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Effective field theory approach to structure functions at small x_{Bj}

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Abstract. We relate the structure functions of deep inelastic lepton–nucleon scattering to current–current correlation functions in a Euclidean field theory depending on a parameter r. The r-dependent Hamiltonian of the theory is $P^0 - (1-r)P^3$, with P^0 the usual Hamiltonian and P^3 the third component of the momentum operator. We show that a small x_{Bj} in the structure functions corresponds to the small r limit of the effective theory. We argue that for $r \to 0$ there is a critical regime of the theory where simple scaling relations should hold. We show that in this framework Regge behaviour of the structure functions obtained with the hard pomeron ansatz corresponds to a scaling behaviour of the matrix elements in the effective theory where the intercept of the hard pomeron appears as a critical index. Explicit expressions for various analytic continuations of the structure functions and matrix elements are given as well as path integral representations for the matrix elements in the effective theory. Our aim is to provide a framework for truly non-perturbative calculations of the structure functions at small x_{Bj} for arbitrary Q^2.

1 Introduction

In this article we shall discuss the small x_{Bj} behaviour of the structure functions of deep inelastic lepton–nucleon scattering (DIS). The findings of the experiments H1 and ZEUS at HERA (for recent summaries see [1–3]) have brought this topic to the forefront of theoretical interest. Soon more data will come from HERA2. There are numerous suggestions for the theoretical description of the small x_{Bj} behaviour of the structure functions; see for instance [4–6]. Let us just mention a few of these approaches with representative references.

As the first group of approaches let us mention the ones based on perturbative QCD, which allows one to derive evolution equations for the structure functions. It should be kept in mind that already in the derivation of these evolution equations one has to make various assumptions and their practical use involves further approximations.

Most popular and widely used is the DGLAP equation [7] to calculate the evolution of the structure functions with Q^2 (see for instance [1, 3, 8]). Improvements of the DGLAP method in fixed order in the strong coupling parameter α_{s}, involving resummations to all orders in α_{s}, have recently been proposed [9]. Another time-honoured approach is based on the BFKL equation [10] which is described in detail for instance in [11]. However, very large higher-order corrections have been found in this approach [12]. Different recipes for dealing with this problem have been proposed [13].

Other approaches make more assumptions and could be called QCD-based or -inspired models. Very popular at the moment are dipole models [14, 15]. Other approaches are based on the semiclassical approximation [6, 16–18] and on the colour glass condensate idea [19, 20].

Quite a different type of approach is based on Regge theory. It was shown in [21, 22] that the small x_{Bj} behaviour of the structure functions can be well described using two pomerons, a hard and a soft one. Both pomerons are assumed to behave like simple Regge poles with linear trajectories.

In this article we continue the investigations of the approach [23] where the behaviour of the structure functions at small x_{Bj} is related to that of matrix elements in an effective Euclidean field theory. In [23] this was explored for a model scalar field theory and it was argued that the limit $x_{\mathrm{Bj}} \to 0$ corresponds to critical behaviour in the effective theory. Here we extend these considerations to the case of QCD. The aim of this approach is to provide a framework where the small x_{Bj} behaviour of the structure functions can be calculated from first principles using truly non-perturbative methods, for instance lattice methods.

Our article is organised as follows. In Sect. 2 we discuss kinematics, the reduced matrix elements free of kinematical singularities and the analytic continuation from the real to the imaginary ν-axis. Section 3 deals with the Deser–Gilbert–Sudarshan (DGS) representation which we use for further analytic continuations. In Sect. 4 we discuss our effective Hamiltonians and Lagrangians for QCD, both

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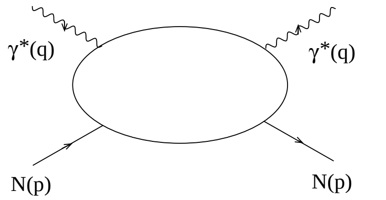

Fig. 1. Virtual Compton scattering on a nucleon

in Minkowski and Euclidean space. Section 5 contains phenomenological applications and Sect. 6 our conclusions.

2 Kinematics and analytic continuation in the ν-plane

In this section we recall first some definitions and results from DIS. The central object of our study is the forward virtual Compton scattering amplitude for nucleons. The absorptive part of this amplitude gives for real ν the measurable cross sections of DIS. Our theoretical investigations will concentrate on the amplitude at imaginary values of ν, from which nevertheless we can obtain information on these cross sections.

2.1 Kinematics

We study the forward virtual Compton scattering amplitude (Fig. 1)

$$\gamma^*(q) + N(p) \to \gamma^*(q) + N(p), \tag{2.1}$$

where N stands for proton or neutron. We consider only spacelike virtual photons, $q^2 = -Q^2 < 0$, and the amplitude averaged over the nucleon spin. The familiar Feynman amplitude for (2.1) is

$$\begin{aligned} T^{\rm F}_{\mu\nu}(p,q) = {} & \frac{\rm i}{2\pi M}\int {\rm d}^4x {\rm e}^{{\rm i}qx} \\ & \times \frac{1}{2}\sum_{\rm spins}\langle N(p)|T^* J_\mu(x)J_\nu(0)|N(p)\rangle. \end{aligned} \tag{2.2}$$

Here M is the nucleon mass, $J_\mu(x)$ is the hadronic part of the electromagnetic current, and T^* indicates the covariant version of the T-product (see for instance [24]). It is understood that only the connected part of the matrix element is taken. All our conventions on kinematics follow [25].

In the following we shall, however, not work with (2.2) but with the retarded amplitude

$$\begin{aligned} T^{\rm ret}_{\mu\nu}(p,q) = {} & \frac{\rm i}{2\pi M}\int {\rm d}^4x {\rm e}^{{\rm i}qx} \\ & \times \frac{1}{2}\sum_{\rm spins}\langle N(p)|\theta(x^0)[J_\mu(x),J_\nu(0)]_{\rm cov}|N(p)\rangle, \end{aligned} \tag{2.3}$$

where we define

$$\theta(x^0)[J_\mu(x),J_\nu(0)]_{\rm cov} = T^*(J_\mu(x)J_\nu(0)) - J_\nu(0)J_\mu(x). \tag{2.4}$$

The standard $W_{\mu\nu}$ tensor of DIS is

$$W_{\mu\nu}(p,q) = \frac{1}{4\pi M}\int {\rm d}^4x {\rm e}^{{\rm i}qx}\mathcal{M}_{\mu\nu}(x,p), \tag{2.5}$$

where

$$\mathcal{M}_{\mu\nu}(x,p) = \frac{1}{2}\sum_{\rm spins}\langle N(p)|J_\mu(x)J_\nu(0)|N(p)\rangle. \tag{2.6}$$

The expansion of $W_{\mu\nu}$ in terms of the structure functions $W_{1,2}$ reads, for $pq > 0$,

$$\begin{aligned} W_{\mu\nu}(p,q) = {} & \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(\nu,Q^2) \\ & + \frac{1}{M^2}\left(p_\mu - \frac{(pq)q_\mu}{q^2}\right)\left(p_\nu - \frac{(pq)q_\nu}{q^2}\right) W_2(\nu,Q^2), \\ & \nu = pq/M, \quad Q^2 = -q^2. \end{aligned} \tag{2.7}$$

In the usual way we define the structure functions $F_{1,2,{\rm L}}$ by

$$\begin{aligned} F_1(x_{\rm Bj},Q^2) &= 2MW_1(\nu,Q^2), \\ F_2(x_{\rm Bj},Q^2) &= \nu W_2(\nu,Q^2), \\ F_L(x_{\rm Bj},Q^2) &= F_2(x_{\rm Bj},Q^2) \\ & \quad - \left(1 + \frac{4M^2}{Q^2}x_{\rm Bj}^2\right)^{-1} x_{\rm Bj}F_1(x_{\rm Bj},Q^2), \\ x_{\rm Bj} &= \frac{Q^2}{2M\nu}, \end{aligned} \tag{2.8}$$

and the transverse and longitudinal cross sections

$$\begin{aligned} \frac{K}{4\pi^2\alpha}\sigma_{\rm T}(\nu,Q^2) &= W_1(\nu,Q^2) = \frac{1}{2M}F_1(x_{\rm Bj},Q^2), \\ \frac{K}{4\pi^2\alpha}\sigma_{\rm L}(\nu,Q^2) &= \frac{\nu^2+Q^2}{Q^2}W_2(\nu,Q^2) - W_1(\nu,Q^2) \\ &= \frac{1}{2Mx_{\rm Bj}}\left(1 + \frac{4M^2}{Q^2}x_{\rm Bj}^2\right) F_{\rm L}(x_{\rm Bj},Q^2), \\ K &= \nu - \frac{Q^2}{2M}. \end{aligned} \tag{2.9}$$

The expansions for $T^{\rm F}_{\mu\nu}$ (2.2) and $T^{\rm ret}_{\mu\nu}$ (2.3) read

$$\begin{aligned} T^{\rm F,ret}_{\mu\nu}(p,q) = {} & \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) T^{\rm F,ret}_1(\nu,Q^2) \qquad (2.10) \\ & + \frac{1}{M^2}\left(p_\mu - \frac{(pq)q_\mu}{q^2}\right)\left(p_\nu - \frac{(pq)q_\nu}{q^2}\right) T^{\rm F,ret}_2(\nu,Q^2), \end{aligned}$$

where now $-\infty < pq < \infty$. For Q^2 fixed, $T^{\rm F}_j(\nu,Q^2)$ and $T^{\rm ret}_j(\nu,Q^2)$ $(j = 1,2)$, are limits of functions $T_j(\nu,Q^2)$ analytic in the cut ν-plane as shown in Fig. 2. The position of the nucleon poles is $\nu = \pm Q^2/(2M)$; the cuts start at

$$\nu = \pm\nu_t = \pm[Q^2 + (M+m_\pi)^2 - M^2]/(2M), \tag{2.11}$$

where m_π is the pion mass. We have, for real ν and $j = 1,2$,

$$T^{\rm F}_j(\nu,Q^2) = \lim_{\epsilon\to+0} T_j(\nu(1+{\rm i}\epsilon),Q^2), \tag{2.12}$$

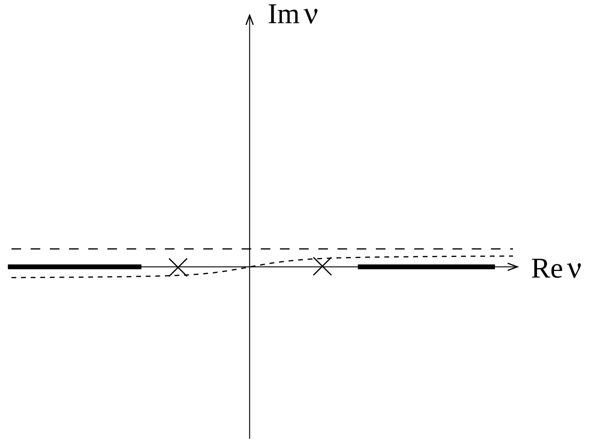

Fig. 2. The ν-plane with the position of the nucleon poles ($\times$) and the cuts. The functions $T_j^{\mathrm{F}}(\nu, Q^2)$ $(j = 1, 2)$ are obtained along the short dashed line, $T_j^{\mathrm{ret}}(\nu, Q^2)$ along the long dashed line

$$T_j^{\mathrm{ret}}(\nu, Q^2) = \lim_{\epsilon\to+0} T_j(\nu + \mathrm{i}\epsilon, Q^2), \tag{2.13}$$

$$\mathrm{Im}\, T_j^{\mathrm{F}}(\nu, Q^2) = \theta(\nu) W_j(\nu, Q^2) + \theta(-\nu) W_j(-\nu, Q^2), \tag{2.14}$$

$$\mathrm{Im}\, T_j^{\mathrm{ret}}(\nu, Q^2) = \theta(\nu) W_j(\nu, Q^2) - \theta(-\nu) W_j(-\nu, Q^2). \tag{2.15}$$

For our purpose it is convenient to define in addition the following scalar amplitudes:

$$T_a^{\mathrm{ret}}(\nu, Q^2) = -g^{\mu\nu} T_{\mu\nu}^{\mathrm{ret}}(p, q) \tag{2.16}$$
$$= 3T_1^{\mathrm{ret}}(\nu, Q^2) - \frac{\nu^2 + Q^2}{Q^2} T_2^{\mathrm{ret}}(\nu, Q^2),$$

$$T_b^{\mathrm{ret}}(\nu, Q^2) = M^{-2} p^\mu p^\nu T_{\mu\nu}^{\mathrm{ret}}(p, q) \tag{2.17}$$
$$= -\frac{\nu^2 + Q^2}{Q^2} T_1^{\mathrm{ret}}(\nu, Q^2) + \left(\frac{\nu^2 + Q^2}{Q^2}\right)^2 T_2^{\mathrm{ret}}(\nu, Q^2).$$

In a similar way we define for real ν the functions $T_{a,b}^{\mathrm{F}}$ and $W_{a,b}$ and for complex ν the functions $T_{a,b}(\nu, Q^2)$. The latter are analytic in the cut ν-plane and have as boundary values for $\mathrm{Im}\nu \to +0$ the functions $T_{a,b}^{\mathrm{ret}}(\nu, Q^2)$.

2.2 Analytic continuation in the ν-plane

We are interested in the behaviour of $W_{1,2}(\nu, Q^2)$ for fixed $Q^2 > 0$ and $\nu \to \infty$. Instead of investigating the structure functions $W_{1,2}$ directly we shall study first the behaviour of the functions $T_{1,2}(\nu, Q^2)$ or equivalently $T_{a,b}(\nu, Q^2)$ for large imaginary ν, that is for $\nu = i\eta$ with $\eta \to \infty$. Then we use the Phragmén–Lindelöf theorem (see theorem 5.64 of [26]) to relate the behaviour of the amplitudes for large real and imaginary values of ν. This has been discussed in detail in Appendix A of [23].

We follow now the same strategy as in [23]. We work in the rest system of the nucleon and choose the direction of $\mathbf{q}$ as third axis of the coordinate system:

$$p = \begin{pmatrix} M \\ \mathbf{0} \end{pmatrix}, \quad q = \begin{pmatrix} \nu \\ \mathbf{e}_3\sqrt{\nu^2 + Q^2} \end{pmatrix}. \tag{2.18}$$

Using rotational symmetry and the same arguments as in [23], we can represent the functions $T_{a,b}(\nu, Q^2)$ for $\mathrm{Im}\ \nu \geq 0$ as follows:

$$T_a(\nu, Q^2) = -\frac{1}{M\sqrt{\nu^2 + Q^2}}$$
$$\times \int_0^\infty \mathrm{d}x^0 \int_{-\infty}^\infty \mathrm{d}x^3 x^3 \exp[\mathrm{i}x^0\nu - \mathrm{i}x^3\sqrt{\nu^2 + Q^2}]$$
$$\times \frac{1}{2}\sum_{\mathrm{spins}} \langle N(p)|(-g^{\mu\nu})\theta(x^0)$$
$$\times [J_\mu(x^3\mathbf{e}_3, x^0), J_\nu(0)]_{\mathrm{cov}}|N(p)\rangle, \tag{2.19}$$

$$T_b(\nu, Q^2) = -\frac{1}{M\sqrt{\nu^2 + Q^2}}$$
$$\times \int_0^\infty \mathrm{d}x^0 \int_{-\infty}^\infty \mathrm{d}x^3 x^3 \exp[\mathrm{i}x^0\nu - \mathrm{i}x^3\sqrt{\nu^2 + Q^2}]$$
$$\times \frac{1}{2}\sum_{\mathrm{spins}} \langle N(p)|\theta(x^0)$$
$$\times [J_0(x^3\mathbf{e}_3, x^0), J_0(0)]_{\mathrm{cov}}|N(p)\rangle. \tag{2.20}$$

For $\nu = \mathrm{i}\eta$ and $\eta > Q$ we get

$$T_a(\mathrm{i}\eta, Q^2) = \frac{\mathrm{i}}{M\sqrt{\eta^2 - Q^2}}$$
$$\times \int_0^\infty \mathrm{d}x^0 \int_{-\infty}^\infty \mathrm{d}x^3 x^3 \exp[-x^0\eta + x^3\sqrt{\eta^2 - Q^2}]$$
$$\times \frac{1}{2}\sum_{\mathrm{spins}} \langle N(p)|(-g^{\mu\nu})\theta(x^0)$$
$$\times [J_\mu(x^3\mathbf{e}_3, x^0), J_\nu(0)]_{\mathrm{cov}}|N(p)\rangle, \tag{2.21}$$

$$T_b(\mathrm{i}\eta, Q^2) = \frac{\mathrm{i}}{M\sqrt{\eta^2 - Q^2}}$$
$$\times \int_0^\infty \mathrm{d}x^0 \int_{-\infty}^\infty \mathrm{d}x^3 x^3 \exp[-x^0\eta + x^3\sqrt{\eta^2 - Q^2}]$$
$$\times \frac{1}{2}\sum_{\mathrm{spins}} \langle N(p)|\theta(x^0)$$
$$\times [J_0(x^3\mathbf{e}_3, x^0), J_0(0)]_{\mathrm{cov}}|N(p)\rangle. \tag{2.22}$$

The commutators in (2.19)–(2.22) vanish for $|x^3| > x^0$ and thus the effective integration range is $|x^3| \leq x^0$. This makes the integrands in (2.19) and (2.20) exponentially damped for $\mathrm{Im}\nu > 0$ and represents the standard way of analytic continuaton of $T_{a,b}^{\mathrm{ret}}(\nu, Q^2)$ into the upper half ν-plane. However, the singularities on the lightcone $|x^3| = x^0$ make it advisable to keep the x^3 integrals in (2.19)–(2.22) as running from $-\infty$ to $+\infty$. From $T_{a,b}(\mathrm{i}\eta, Q^2)$ we can derive $T_{1,2}(\mathrm{i}\eta, Q^2)$ using (2.16) and (2.17).

In the next section we will derive another representation for $T_{1,2}(\mathrm{i}\eta, Q^2)$ in which the integration path avoids the lightcone.

3 The matrix element $\mathcal{M}_{\mu\nu}$

In this section we list general properties of the matrix element $\mathcal{M}_{\mu\nu}(x, p)$ of (2.6) and discuss its analyticity properties using the Deser–Gilbert–Sudarshan (DGS) representation [27, 28].

3.1 General properties of $\mathcal{M}_{\mu\nu}(x,p)$

Hermiticity of the electromagnetic current gives

$$\mathcal{M}_{\mu\nu}(x,p)^* = \mathcal{M}_{\nu\mu}(-x,p). \quad (3.1)$$

Time reversal invariance of QCD requires

$$\mathcal{M}_{\mu\nu}(x,p) = \mathcal{M}^{\nu\mu}(x',p'), \quad (3.2)$$

where $x'^{\mu} = x_\mu, p'^{\mu} = p_\mu$.
Current conservation implies

$$\frac{\partial}{\partial x^\mu}\mathcal{M}^{\mu\nu}(x,p) = 0, \qquad \frac{\partial}{\partial x^\nu}\mathcal{M}^{\mu\nu}(x,p) = 0. \quad (3.3)$$

For $x^2 < 0$ the currents $J_\mu(x)$ and $J_\nu(0)$ in (2.6) commute, which leads to

$$\mathcal{M}_{\mu\nu}(x,p) = \mathcal{M}_{\nu\mu}(-x,p), \quad \text{for } x^2 < 0. \quad (3.4)$$

The expansion of $\mathcal{M}_{\mu\nu}(x,p)$ in terms of two causal scalar functions which are free from kinematical singularities and where (3.1) to (3.4) are satisfied automatically [29] reads

$$\begin{aligned}\mathcal{M}_{\mu\nu}(x,p) = {} & [g_{\mu\nu}\Box - \partial_\mu\partial_\nu]\mathcal{M}_1(xp,x^2) \\ & + [p_\mu p_\nu \Box - (p\partial)(p_\mu\partial_\nu + p_\nu\partial_\mu) + g_{\mu\nu}(p\partial)^2] \\ & \times \mathcal{M}_2(xp,x^2). \end{aligned} \quad (3.5)$$

The invariant functions $\mathcal{M}_j$ $(j=1,2)$ satisfy

$$\mathcal{M}_j(xp,x^2) = \mathcal{M}_j^*(-xp,x^2) \quad (3.6)$$

and

$$\mathcal{M}_j(xp,x^2) = \mathcal{M}_j(-xp,x^2), \quad \text{for } x^2 < 0. \quad (3.7)$$

3.2 Representations of amplitudes in terms of $\mathcal{M}_{1,2}$

Inserting (3.5) into (2.5) we obtain the structure functions $W_{1,2}$ of (2.7) in terms of the Fourier transforms of $\mathcal{M}_{1,2}$:

$$\begin{aligned} W_1(\nu,Q^2) = {} & -\frac{Q^2}{4\pi M}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx}\mathcal{M}_1(xp,x^2) \\ & + \frac{(pq)^2}{4\pi M}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx}\mathcal{M}_2(xp,x^2), \end{aligned} \quad (3.8)$$

$$W_2(\nu,Q^2) = \frac{MQ^2}{4\pi}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx}\mathcal{M}_2(xp,x^2). \quad (3.9)$$

Note that in (3.9) the kinematic zero of $W_2(\nu,Q^2)$ at $Q^2 = 0$ is made explicit, as it should be. For the cross sections (2.9) we get

$$\begin{aligned}\frac{K}{4\pi^2\alpha}\sigma_{\rm T}(\nu,Q^2) = {} & \frac{1}{4\pi M}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx} \\ & \times [M^2\nu^2\mathcal{M}_2(xp,x^2) - Q^2\mathcal{M}_1(xp,x^2)], \end{aligned} \quad (3.10)$$

$$\begin{aligned}\frac{K}{4\pi^2\alpha}\sigma_{\rm L}(\nu,Q^2) = {} & \frac{Q^2}{4\pi M}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx} \\ & \times [M^2\mathcal{M}_2(xp,x^2) + \mathcal{M}_1(xp,x^2)]. \end{aligned} \quad (3.11)$$

Inserting (3.5) into (2.3) and using (2.10) and rotational symmetry in the nucleon rest frame, we get

$$\begin{aligned} T_1^{\rm ret}(\nu,Q^2) = {} & -\frac{1}{M\sqrt{\nu^2+Q^2}} \\ & \times \int_0^\infty \mathrm{d}x^0 \int_{-\infty}^\infty \mathrm{d}x^3 x^3 \exp[\mathrm{i}x^0\nu - \mathrm{i}x^3\sqrt{\nu^2+Q^2}] \\ & \times \Big\{ - Q^2[\mathcal{M}_1(xp,x^2) - \mathcal{M}_1^*(xp,x^2)] \\ & + M^2\nu^2[\mathcal{M}_2(xp,x^2) - \mathcal{M}_2^*(xp,x^2)]\Big\}, \end{aligned} \quad (3.12)$$

$$\begin{aligned} T_2^{\rm ret}(\nu,Q^2) = {} & -\frac{MQ^2}{\sqrt{\nu^2+Q^2}} \\ & \times \int_0^\infty \mathrm{d}x^0 \int_{-\infty}^\infty \mathrm{d}x^3 x^3 \exp[\mathrm{i}x^0\nu - \mathrm{i}x^3\sqrt{\nu^2+Q^2}] \\ & \times [\mathcal{M}_2(xp,x^2) - \mathcal{M}_2^*(xp,x^2)], \end{aligned} \quad (3.13)$$

where

$$xp = x^0 M, \quad x^2 = (x^0)^2 - (x^3)^2. \quad (3.14)$$

In the following we will make a change of variables as in [23] and replace (x^0, x^3) by (x^0, r), where

$$r = 1 - \frac{x^3}{x^0}. \quad (3.15)$$

We define

$$\widetilde{\mathcal{M}}_j^-(x^0,r) := \mathcal{M}_j(x^0M, (x^0)^2 r(2-r)) \qquad (j=1,2). \quad (3.16)$$

With this we get

$$\begin{aligned} T_1^{\rm ret}(\nu,Q^2) = {} & -\frac{1}{M\sqrt{\nu^2+Q^2}}\int_{-\infty}^\infty \mathrm{d}r(1-r) \\ & \times \int_0^\infty \mathrm{d}x^0 (x^0)^2 \exp[\mathrm{i}x^0(\nu - (1-r)\sqrt{\nu^2+Q^2})] \\ & \times \Big\{ - Q^2[\widetilde{\mathcal{M}}_1^-(x^0,r) - (\widetilde{\mathcal{M}}_1^-(x^0,r))^*] \\ & + M^2\nu^2[\widetilde{\mathcal{M}}_2^-(x^0,r) - (\widetilde{\mathcal{M}}_2^-(x^0,r))^*]\Big\}, \end{aligned} \quad (3.17)$$

$$\begin{aligned} T_2^{\rm ret}(\nu,Q^2) = {} & -\frac{MQ^2}{\sqrt{\nu^2+Q^2}}\int_{-\infty}^\infty \mathrm{d}r(1-r) \\ & \times \int_0^\infty \mathrm{d}x^0 (x^0)^2 \exp[\mathrm{i}x^0(\nu - (1-r)\sqrt{\nu^2+Q^2})] \\ & \times [\widetilde{\mathcal{M}}_2^-(x^0,r) - (\widetilde{\mathcal{M}}_2^-(x^0,r))^*]. \end{aligned} \quad (3.18)$$

3.3 DGS representations for $\mathcal{M}_{1,2}$

For the causal functions $\mathcal{M}_{1,2}(xp, x^2)$ defined in (3.5) we can write down DGS representations [27, 28]. The behaviour of the structure functions for small x_{Bj} as measured in experiments (see Sect. 5 below) suggests that the DGS representation for $\mathcal{M}_2$ needs no subtraction, whereas $\mathcal{M}_1$ needs subtractions. Thus we write

$$\mathcal{M}_2(xp, x^2) = \int_0^\infty \mathrm{d}s \int_{-1}^1 \mathrm{d}\zeta f_2(s,\zeta) \exp(\mathrm{i}\zeta px) \frac{1}{\mathrm{i}} \Delta^+(x,s), \tag{3.19}$$

where

$$\frac{1}{\mathrm{i}}\Delta^+(x,s) = \frac{1}{(2\pi)^3}\int \mathrm{d}^4k \mathrm{e}^{-\mathrm{i}kx}\theta(k^0)\delta(k^2-s)$$
$$= \frac{1}{4\pi^2}\left(\frac{s}{-x^2+\mathrm{i}\epsilon x^0}\right)^{1/2} K_1\left(\sqrt{s(-x^2+\mathrm{i}\epsilon x^0)}\right) \tag{3.20}$$

is one of the usual invariant functions given in terms of the modified Bessel function K_1. In (3.19) f_2 is a function for which (3.6) and (3.7) require

$$f_2(s,\zeta) = f_2(s,-\zeta) = f_2^*(s,\zeta). \tag{3.21}$$

Thus, (3.19) can also be written as

$$\mathcal{M}_2(xp, x^2) = \int_0^\infty \mathrm{d}s \int_{-1}^1 \mathrm{d}\zeta f_2(s,\zeta)\cos(\zeta px)\frac{1}{\mathrm{i}}\Delta^+(x,s). \tag{3.22}$$

For $\mathcal{M}_1$ we write down a subtracted DGS representation:

$$\mathcal{M}_1(xp, x^2) = \int_0^\infty \mathrm{d}s$$
$$\times \left\{ f_1^{(0)}(s) + \int_{-1}^1 \mathrm{d}\zeta f_1(s,\zeta)[\cos(\zeta px) - 1]\right\}$$
$$\times \frac{1}{\mathrm{i}}\Delta^+(x,s), \tag{3.23}$$

where

$$f_1^{(0)}(s) = \left(f_1^{(0)}(s)\right)^*,$$
$$f_1(s,\zeta) = f_1(s,-\zeta) = f_1^*(s,\zeta). \tag{3.24}$$

In [27, 28] the DGS representation is primarily discussed for the matrix element of the commutator and the T-product of two currents. From the DGS representation of the T-product we get immediately the representation for the ordinary products of currents used here.

Let us now switch to the matrix elements $\widetilde{\mathcal{M}}_j^-(x^0, r)$ (3.16) to discuss their behaviour in r for fixed x^0. For $\widetilde{\mathcal{M}}_2^-$ we get from (3.19)–(3.22), for $x^0 > 0$,

$$\widetilde{\mathcal{M}}_2^-(x^0, r) \equiv \widetilde{\mathcal{M}}_2(x^0, r, \epsilon)$$
$$= \int_0^\infty \mathrm{d}s \int_{-1}^1 \mathrm{d}\zeta f_2(s,\zeta)\cos(\zeta M x^0)$$
$$\times \frac{1}{4\pi^2}\frac{s^{1/2}}{x^0}\left(-r(2-r)+\mathrm{i}\epsilon\right)^{-1/2}$$
$$\times K_1\left(s^{1/2}x^0(-r(2-r)+\mathrm{i}\epsilon)^{1/2}\right). \tag{3.25}$$

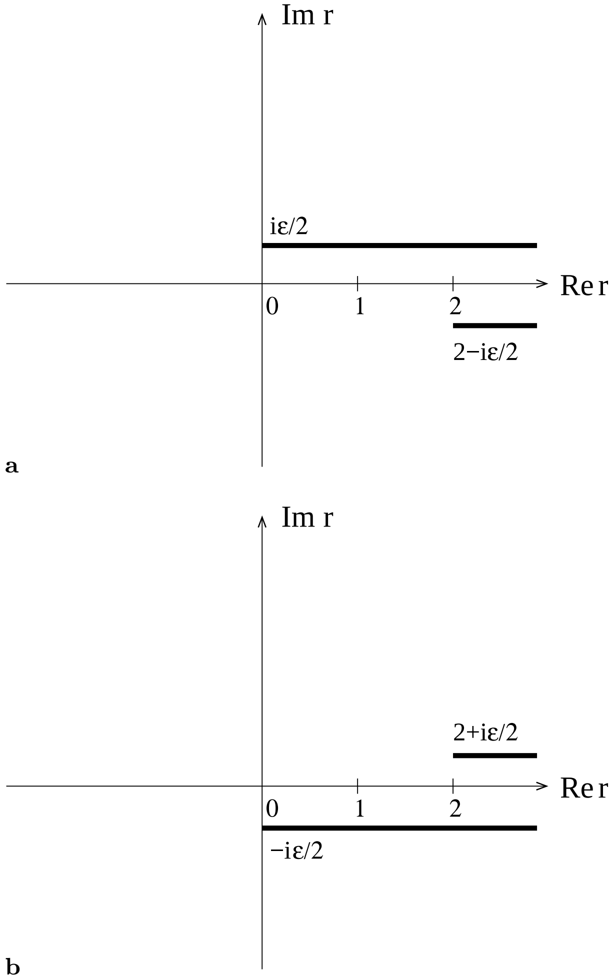

Fig. 3a,b. Cut structure of $\widetilde{\mathcal{M}}(x^0, r, \varepsilon)$ **a** and $\widetilde{\mathcal{M}}^*(x^0, r^*, \varepsilon)$ **b** in the complex r-plane for $0 < \varepsilon << 1$

For clarity we keep in the following discussion the ϵ parameter ($\epsilon > 0$) as an explicit argument. A similar expression is obtained for $\widetilde{\mathcal{M}}_1^-$ from (3.23).

We will now make the assumption that the weight functions $f_j(s,\zeta)$ are sufficiently well behaved, so that the representation (3.25) for $\widetilde{\mathcal{M}}_2(x^0, r, \epsilon)$ and the analogous one for $\widetilde{\mathcal{M}}_1$ allow an analytic continuation in r at fixed x^0 and ϵ. The singularity structure of $\widetilde{\mathcal{M}}_j(x^0, r, \epsilon)$ in the r-plane can then be read off from (3.25). There are two branch points, at $r = 1 \mp (1-\mathrm{i}\epsilon)^{1/2}$, giving, for $\epsilon \ll 1$,

$$r \approx \frac{\mathrm{i}}{2}\epsilon, \quad \text{and} \quad r \approx 2 - \frac{\mathrm{i}}{2}\epsilon. \tag{3.26}$$

We draw the associated cuts to the right (see Fig. 3a). The cut structure of

$$(\widetilde{\mathcal{M}}_j(x^0, r^*, \epsilon))^* = \widetilde{\mathcal{M}}_j(x^0, r, -\epsilon) \tag{3.27}$$

is shown in Fig. 3b. Looking now at (3.17) and (3.18) we see that the r integrations run on the real r-axis from $r = -\infty$ to $r = +\infty$ over $\widetilde{\mathcal{M}}_j(x^0, r, \epsilon)$ and from $r = \infty$ back to $r = -\infty$ over $\widetilde{\mathcal{M}}_j(x^0, r, -\epsilon)$. Deforming the contours

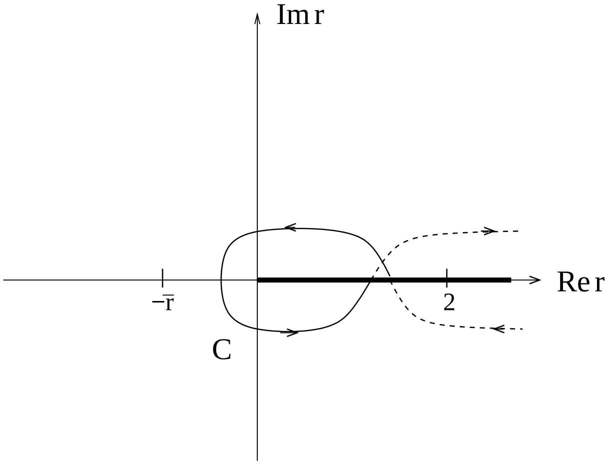

Fig. 4. Cut structure of $\widetilde{\mathcal{M}}_j(x^0, r)$ and the curve C in the complex r-plane. For $\bar{r}$ see (3.34)

slightly we can obtain $T^{\rm ret}_{1,2}$ as integrals over the analytic functions

$$\widetilde{\mathcal{M}}_j(x^0, r) := \widetilde{\mathcal{M}}_j(x^0, r, 0) \tag{3.28}$$

along a curve C (see Fig. 4). The curve C comes from the second sheet, moves onto the first sheet between $r = 0$ and 2, circles $r = 0$ and moves again onto another sheet between $r = 0$ and 2. We get

$$\begin{aligned} T^{\rm ret}_1(\nu, Q^2) &= -\frac{1}{M\sqrt{\nu^2+Q^2}} \int_C {\rm d}r(1-r) \\ &\times \int_0^\infty {\rm d}x^0 (x^0)^2 \exp[{\rm i}x^0(\nu - (1-r)\sqrt{\nu^2+Q^2})] \\ &\times \{-Q^2\widetilde{\mathcal{M}}_1(x^0, r) + M^2\nu^2\widetilde{\mathcal{M}}_2(x^0, r)\}, \end{aligned} \tag{3.29}$$

$$\begin{aligned} T^{\rm ret}_2(\nu, Q^2) &= -\frac{MQ^2}{\sqrt{\nu^2+Q^2}} \int_C {\rm d}r(1-r) \\ &\times \int_0^\infty {\rm d}x^0 (x^0)^2 \exp[{\rm i}x^0(\nu - (1-r)\sqrt{\nu^2+Q^2})] \\ &\times \widetilde{\mathcal{M}}_2(x^0, r). \end{aligned} \tag{3.30}$$

Note that the matrix elements $\widetilde{\mathcal{M}}^-_j(x^0, r)$ of (3.16), for $0 < r < 2$, are the limits of the analytic functions $\widetilde{\mathcal{M}}_j(x^0, r)$ below the cut:

$$\widetilde{\mathcal{M}}^-_j(x^0, r) = \lim_{\epsilon\to+0} \widetilde{\mathcal{M}}_j(x^0, r - {\rm i}\epsilon) \quad (0 < r < 2). \tag{3.31}$$

We can now use (3.29) and (3.30) to perform the analytic continuation in ν to the upper half ν-plane in an alternative way to that of Sect. 2.2. We move the curve C arbitrarily close to the positive real r-axis, $r > 0$, and use the inequalities (A.1) of [23] to show that the exponentials in (3.29) and (3.30) are damped, for ${\rm Im}\nu > 0$. We are interested in $T_{1,2}$ for large imaginary ν. With $\nu = {\rm i}\eta$, $\eta > Q$, the exponentials can be written as

$$\exp[{\rm i}x^0(\nu - (1-r)\sqrt{\nu^2+Q^2})]|_{\nu={\rm i}\eta} = \exp[-x^0/\bar{x}(\eta, r)], \tag{3.32}$$

$$\bar{x}(\eta, r) = [\eta - (1-r)\sqrt{\eta^2 - Q^2}]^{-1} = \frac{1}{\eta}\frac{1+\bar{r}}{r+\bar{r}}, \tag{3.33}$$

$$\bar{r} = \frac{Q^2}{2\eta^2}\frac{2}{1 - \dfrac{Q^2}{\eta^2} + \sqrt{1 - \dfrac{Q^2}{\eta^2}}}. \tag{3.34}$$

With (3.32) we get for the analytically continued functions $T_{1,2}$ from (3.29) and (3.30)

$$\begin{aligned} T_1({\rm i}\eta, Q^2) &= -\frac{\rm i}{M\sqrt{\eta^2-Q^2}} \int_C {\rm d}r(1-r) \int_0^\infty {\rm d}x^0 (x^0)^2 \\ &\times \exp[-x^0/\bar{x}(\eta, r)] \\ &\times [Q^2\widetilde{\mathcal{M}}_1(x^0, r) + M^2\eta^2\widetilde{\mathcal{M}}_2(x^0, r)], \end{aligned} \tag{3.35}$$

$$\begin{aligned} T_2({\rm i}\eta, Q^2) &= \frac{{\rm i}MQ^2}{\sqrt{\eta^2-Q^2}} \int_C {\rm d}r(1-r) \int_0^\infty {\rm d}x^0 (x^0)^2 \\ &\times \exp[-x^0/\bar{x}(\eta, r)]\widetilde{\mathcal{M}}_2(x^0, r). \end{aligned} \tag{3.36}$$

We note that for given Q^2 and $\eta > Q$ the characteristic damping length $\bar{x}(\eta, r)$ is positive, for

$$ {\rm Re} r > -\bar{r}. \tag{3.37}$$

We will thus take in the following the curve C in (3.35) and (3.36) to be to the right of $-\bar{r}$ as shown in Fig. 4. In this way we get integral representations of $T_{1,2}$ $({\rm i}\eta, Q^2)$ where the integration path avoids the singularities of the integrand at $r = 0, 2$, which correspond to the lightcone.

The representations (3.35) and (3.36) will be used as a basis for the discussion in the following sections.

Let us now estimate the relevant integration range in r in (3.35) and (3.36), for $\eta \to \infty$. We see from (3.33) that, for fixed r, we have

$$\bar{x}(\eta, r) \sim \eta^{-1}, \quad \text{for} \quad \eta \to \infty. \tag{3.38}$$

Then the factors $\exp(-x^0/\bar{x}(\eta, r))$ will suppress such contributions to the integrals (3.35) and (3.36). Now we can keep the curve C in Fig. 4 at finite fixed values of r except for the region between $r = 0$ and $r = -\bar{r}$ where C has to cross the negative real axis. Taking as a typical value $r = -\bar{r}/2$ we get

$$\bar{x}\left(\eta, -\frac{1}{2}\bar{r}\right) = \frac{2(1+\bar{r})}{\eta\bar{r}} \sim \frac{2\eta}{Q^2}, \quad \text{for} \quad \eta \to \infty. \tag{3.39}$$

Thus, for $\eta \to \infty$ this region, where $\bar{x}(\eta, r)$ becomes very large, will give the main contribution to the integrals (3.35) and (3.36). The behaviour of $T_{1,2}({\rm i}\eta, Q^2)$, for $\eta \to \infty$, is therefore expected to be governed by the behaviour of $\widetilde{\mathcal{M}}_{1,2}(x^0, r)$, for small $|r|$ and large x^0.

4 Effective Hamiltonians and Lagrangians

In this section we shall express the matrix elements $\mathcal{M}_{\mu\nu}$ (3.5) as functional integrals with effective Lagrangians containing r (3.15) as parameter. The procedure is analogous to the one of [23] but the vector nature of the electromagnetic current causes some complications. We work in this section with the Lagrangian of QCD in a general covariant gauge. The case of the temporal gauge is treated in Appendix C.

4.1 Matrix elements in Minkowski and Euclidean space

We start here with the following matrix elements in the nucleon rest frame, supposing always $x^3 = x^0(1-r), x^0 \geq 0$:

$$\widetilde{\mathcal{M}}_a^-(x^0,r) = \frac{1}{2}\sum_{\rm spins}\langle N(p)|(-g^{\mu\nu}) \times J_\mu(x^3\mathbf{e}_3,x^0)J_\nu(0)|N(p)\rangle, \quad (4.1)$$

$$\widetilde{\mathcal{M}}_b^-(x^0,r) = \frac{1}{2}\sum_{\rm spins}\langle N(p)|J_0(x^3\mathbf{e}_3,x^0)J_0(0)|N(p)\rangle. \quad (4.2)$$

The relation of $\widetilde{\mathcal{M}}_{a,b}^-(x^0,r)$ and $\widetilde{\mathcal{M}}_{1,2}^-(x^0,r)$ (3.16) is given in Appendix A. From translational invariance we have

$$J_\mu(x^3\mathbf{e}_3,x^0) = \exp(\mathrm{i}x^0H_r)J_\mu(0)\exp(-\mathrm{i}x^0H_r), \quad (4.3)$$

where

$$H_r = P^0 - (1-r)P^3, \quad (4.4)$$

and P^0, P^3 are the energy and third component of the momentum operator. Note that H_r is positive semidefinite, for $0 < r < 2$. From (4.3) we get

$$\widetilde{\mathcal{M}}_a^-(x^0,r) = \frac{1}{2}\sum_{\rm spins}\langle N(p)|(-g^{\mu\nu}) \times \exp(\mathrm{i}x^0H_r)J_\mu(0)\exp(-\mathrm{i}x^0H_r)J_\nu(0)|N(p)\rangle, \quad (4.5)$$

$$\widetilde{\mathcal{M}}_b^-(x^0,r) = \frac{1}{2}\sum_{\rm spins}\langle N(p)|\exp(\mathrm{i}x^0H_r) \times J_0(0)\exp(-\mathrm{i}x^0H_r)J_0(0)|N(p)\rangle. \quad (4.6)$$

We see that in the effective theory $\widetilde{\mathcal{M}}_{a,b}^-(x^0,r)$ are correlation functions of two currents at purely timelike separation x^0.

We will now keep r fixed with $0 < r < 2$ where H_r is positive semidefinite. This allows us to continue $\widetilde{\mathcal{M}}_{a,b}^-(x^0,r)$ analytically into the lower half x^0-plane and in particular to $x^0 = -\mathrm{i}X_4$, with $X_4 > 0$:

$$\widetilde{\mathcal{M}}_a^-(-\mathrm{i}X_4,r) = \frac{1}{2}\sum_{\rm spins}\langle N(p)|(-g^{\mu\nu}) \times \exp(X_4H_r)J_\mu(0)\exp(-X_4H_r)J_\nu(0)|N(p)\rangle, \quad (4.7)$$

$$\widetilde{\mathcal{M}}_b^-(-\mathrm{i}X_4,r) = \frac{1}{2}\sum_{\rm spins}\langle N(p)|\exp(X_4H_r)J_0(0) \times \exp(-X_4H_r)J_0(0)|N(p)\rangle. \quad (4.8)$$

The matrix elements (4.7) and (4.8) are correlation functions of two currents at purely timelike distance X_4 in a Euclidean field theory with H_r as Euclidean Hamiltonian.

In the following we shall write the matrix elements (4.5) and (4.6) as path integrals in an effective, r-dependent, Minkoswkian theory and (4.7) and (4.8) in an effective Euclidean theory.

4.2 Effective Lagrangian and path integral in Minkowski space

We start with the Lagrangian of QCD in a general covariant gauge

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{2}\xi^{-1}(\partial^\mu G^a_\mu)(\partial^\nu G^a_\nu)\\ &+ \sum_q \bar{q}\left(\frac{\mathrm{i}}{2}\gamma^\mu \overleftrightarrow{D}_\mu - m_q\right)q\\ &+ (\partial^\mu\bar{\phi}^a)(\partial_\mu\phi^a) - gf_{abc}(\partial^\mu\bar{\phi}^a)G^b_\mu\phi^c. \end{aligned}\quad (4.9)$$

Here q denotes the quark fields, ϕ^a the Fadeev–Popov fields, G^a_μ the gluon potentials and $G^a_{\mu\nu}$ the gluon field strength tensor,

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - gf_{abc}G^b_\mu G^c_\nu. \quad (4.10)$$

The coupling constant is g, the quark masses are m_q, and D_μ is the covariant derivative

$$D_\mu q = \left(\partial_\mu + \mathrm{i}gG^a_\mu\frac{1}{2}\lambda_a\right)q. \quad (4.11)$$

All quantities in (4.9) are the unrenormalised ones.

It is now a straightforward exercise to derive from (4.9) the canonical momenta $\mathit{\Pi}$, the Hamiltonian density, the Hamiltonian P^0, the third component of the momentum P^3 and the effective Hamiltonian H_r (4.4). The details are given in Appendix B. Here we list the result:

$$H_r = \int \mathrm{d}^3x\,\mathcal{H}_r, \quad (4.12)$$

$$\begin{aligned}\mathcal{H}_r = &-\frac{1}{2}\xi\mathit{\Pi}_{G^{a0}}\mathit{\Pi}_{G^{a0}} - \mathit{\Pi}_{G^{a0}}(\partial_jG^{aj} - (1-r)\partial_3G^{a0})\\ &+ \frac{1}{2}\mathit{\Pi}_{G^{aj}}\mathit{\Pi}_{G^{aj}} + \mathit{\Pi}_{G^{aj}}(-\partial_jG^{a0} + gf_{abc}G^{b0}G^{cj}\\ &+ (1-r)\partial_3G^{aj}) + \frac{1}{4}G^a_{jk}G^{ajk}\\ &+ \sum_q \bar{q}\left(-\frac{\mathrm{i}}{2}\gamma^j\overleftrightarrow{\partial}_j + \frac{\mathrm{i}}{2}\gamma^0(1-r)\overleftrightarrow{\partial}_3\right.\\ &\left.+ g\gamma^\mu G^a_\mu\frac{1}{2}\lambda_a + m_q\right)q\\ &+ \mathit{\Pi}_{\phi^a}\mathit{\Pi}_{\bar{\phi}^a} + \mathit{\Pi}_{\phi^a}(gf_{abc}G^{b0}\phi^c + (1-r)\partial_3\phi^a)\\ &+ (1-r)(\partial_3\bar{\phi}^a)\mathit{\Pi}_{\bar{\phi}^a} + (\partial_j\bar{\phi}^a)\partial_j\phi^a\\ &+ gf_{abc}(\partial_j\bar{\phi}^a)G^{bj}\phi^c. \end{aligned}\quad (4.13)$$

Here and in the following Latin indices $j, k, \ldots$ run from 1 to 3. From the effective Hamiltonian density (4.13) we get in the standard way the effective Lagrangian density:

$$
\begin{aligned}
\mathcal{L}_r = & -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - (1-r)G^{a0j}\partial_3 G^{aj} \\
& + \frac{1}{2}(1-r)^2(\partial_3 G^{aj})\partial_3 G^{aj} \\
& - \frac{1}{2}\xi^{-1}(\partial_\mu G^{a\mu} - (1-r)\partial_3 G^{a0})(\partial_\nu G^{a\nu} \\
& - (1-r)\partial_3 G^{a0}) \\
& + \sum_q \bar{q}\left(\frac{\mathrm{i}}{2}\gamma^\mu \overleftrightarrow{D}_\mu - m_q - (1-r)\frac{\mathrm{i}}{2}\gamma^0 \overleftrightarrow{\partial}_3\right) q \\
& + (\partial^\mu \bar{\phi}^a)\partial_\mu \phi^a - g f_{abc}(\partial_\mu \bar{\phi}^a)G^{b\mu}\phi^c \\
& - (1-r)\left((\partial_3\bar{\phi}^a)\dot{\phi}^a + \dot{\bar{\phi}}^a\partial_3\phi^a\right) \\
& + (1-r)\left(\partial_3\bar{\phi}^a\right) g f_{abc} G^{b0}\phi^c \\
& + (1-r)^2\left(\partial_3\bar{\phi}^a\right)\partial_3\phi^a. \qquad (4.14)
\end{aligned}
$$

Here and in the following we use the term "effective" for the r-dependent theory. We emphasise that our procedure does not imply approximations like integrating out some modes etc. The r-dependent theory is as good as the original one for the calculation of our matrix elements and we hope that it will be more effective for the study of the small $x_{\rm Bj}$ limit of the structure functions.

We can now give the path integral representation of $\widetilde{\mathcal{M}}^-_{a,b}(x^0,r)$ of (4.5) and (4.6) in the effective theory described by (4.14). The procedure is analogous to the one described in detail in [23]. Let $\psi_N(x)$ be an interpolating field operator for the nucleon with normalisation such that

$$\langle 0|\psi_N(x)|N(p,s)\rangle = \mathrm{e}^{-\mathrm{i}px}u_s(p) \quad (s=\pm 1/2). \qquad (4.15)$$

According to the LSZ formalism [30] we define

$$A_s(p,t) = \int_{x^0=t} \mathrm{d}^3x\,\bar{u}_s(p)\mathrm{e}^{\mathrm{i}px}\gamma^0\psi_N(x). \qquad (4.16)$$

In the sense of the weak limit we have

$$
\begin{aligned}
\lim_{t\to-\infty} A_s(p,t) &= A_s^{\rm in}(p), \\
\lim_{t\to+\infty} A_s(p,t) &= A_s^{\rm out}(p), \qquad (4.17)
\end{aligned}
$$

where $A_s^{\rm in}(p)$ and $A_s^{\rm out}(p)$ are the annihilation operators for incoming and outgoing nucleons. In the standard way we obtain now for the matrix elements

$$
\begin{aligned}
\widetilde{\mathcal{M}}^-_a(x^0,r) = & \lim_{\substack{t_i\to-\infty \\ t_f\to+\infty}} \exp[\mathrm{i}M(t_f-t_i)] \\
& \times Z^{-1}\int \mathcal{D}(G,q,\bar{q},\phi,\bar{\phi})\exp\left[\mathrm{i}\int \mathrm{d}^4x\mathcal{L}_r(x)\right] \\
& \times \frac{1}{2}\sum_s a_s(p,t_f)(-g^{\mu\nu})j_\mu(\mathbf{0},x^0)j_\nu(0)a_s^\dagger(p,t_i), \qquad (4.18)
\end{aligned}
$$

$$Z = \int \mathcal{D}(G,q,\bar{q},\phi,\bar{\phi})\exp\left[\mathrm{i}\int \mathrm{d}^4x\mathcal{L}_r(x)\right], \qquad (4.19)$$

$$
\begin{aligned}
\widetilde{\mathcal{M}}^-_b(x^0,r) = & \lim_{\substack{t_i\to-\infty \\ t_f\to+\infty}} \exp[\mathrm{i}M(t_f-t_i)] \\
& \times Z^{-1}\int \mathcal{D}(G,q,\bar{q},\phi,\bar{\phi})\exp\left[\mathrm{i}\int \mathrm{d}^4x\mathcal{L}_r(x)\right] \\
& \times \frac{1}{2}\sum_s a_s(p,t_f)j_0(\mathbf{0},x^0)j_0(0)a_s^\dagger(p,t_i). \qquad (4.20)
\end{aligned}
$$

Here p is always the nucleon momentum in the rest system (see (2.18)) and $a_s, a_s^\dagger, j_\mu$ are obtained by replacing in $A_s, A_s^\dagger, J_\mu$ the quark field operators by the corresponding Grassmann variables. Of course, also in $\mathcal{L}_r(x)$ quark, ghost and gluon field operators have to be replaced by Grassmann variables and classical gluon fields, respectively. With (4.18)–(4.20) we have the path integral representation of the matrix elements $\widetilde{\mathcal{M}}^-_{a,b}$ as timelike correlation functions of the currents in the effective, r-dependent Minkowskian theory.

4.3 Effective Lagrangian and path integral in Euclidean space

Here we derive the path integral representation for the matrix elements (4.7) and (4.8) in the Euclidean effective theory. Points in Euclidean space are denoted by $X = (\mathbf{X}, X_4)$. Our Euclidean γ-matrices are

$$
\begin{aligned}
\gamma_{\mathrm{E}j} &= -\mathrm{i}\gamma^j \quad (j=1,2,3), \\
\gamma_{\mathrm{E}4} &= \gamma^0. \qquad (4.21)
\end{aligned}
$$

We perform now a rotation to Euclidean space starting from the effective Hamiltonian (4.12) and (4.13). The details are given in Appendix D. The result for the effective Lagrangian density in Euclidean space is

$$
\begin{aligned}
\mathcal{L}_{\mathrm{E},r} = & \frac{1}{2}\xi^{-1}[\partial_\mu G^a_{\mathrm{E}\mu} + \mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}4}] \\
& \times [\partial_\nu G^a_{\mathrm{E}\nu} + \mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}4}] \\
& + \frac{1}{2}[G^a_{\mathrm{E}4j} + \mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}j}][G^a_{\mathrm{E}4j} + \mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}j}] \\
& + \frac{1}{4}G^a_{\mathrm{E}jk}G^a_{\mathrm{E}jk} \\
& + \sum_q \bar{q}_{\mathrm{E}}\left(\frac{1}{2}\gamma_{E\mu}\overleftrightarrow{\partial}_\mu + \frac{\mathrm{i}}{2}\gamma_{\mathrm{E}4}(1-r)\overleftrightarrow{\partial}_3\right. \\
& \left. + \mathrm{i}g\gamma_{\mathrm{E}\mu}G^a_{\mathrm{E}\mu}\frac{1}{2}\lambda_a + m_q\right) q_{\mathrm{E}} \\
& + (\partial_4\bar{\phi}^a_{\mathrm{E}} + \mathrm{i}(1-r)\partial_3\bar{\phi}^a_{\mathrm{E}})(\partial_4\phi^a_{\mathrm{E}} - g f_{abc}G^b_{\mathrm{E}4}\phi^c_{\mathrm{E}} \\
& + \mathrm{i}(1-r)\partial_3\phi^a_{\mathrm{E}}) \\
& + (\partial_j\bar{\phi}^a_{\mathrm{E}})\partial_j\phi^a_{\mathrm{E}} - g f_{abc}(\partial_j\bar{\phi}^a_{\mathrm{E}})G^b_{\mathrm{E}j}\phi^c_{\mathrm{E}}. \qquad (4.22)
\end{aligned}
$$

For the following it is convenient to split $\mathcal{L}_{\mathrm{E},r}$ into the quadratic and the interaction term:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{E},r} &= \mathcal{L}^{(0)}_{\mathrm{E},r} + \mathcal{L}^{\rm Int}_{\mathrm{E},r}, \qquad (4.23) \\
\mathcal{L}^{(0)}_{\mathrm{E},r} &= \frac{1}{2}\xi^{-1}[\partial_\mu G^a_{\mathrm{E}\mu} + \mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}4}] \\
& \quad \times [\partial_\nu G^a_{\mathrm{E}\nu} + \mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}4}]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}[\partial_4 G^a_{\mathrm{E}j}-\partial_j G^a_{\mathrm{E}4}+\mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}j}]\\
&\times[\partial_4 G^a_{\mathrm{E}j}-\partial_j G^a_{\mathrm{E}4}+\mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}j}]\\
&+\frac{1}{4}[\partial_j G^a_{\mathrm{E}k}-\partial_k G^a_{\mathrm{E}j}][\partial_j G^a_{\mathrm{E}k}-\partial_k G^a_{\mathrm{E}j}]\\
&+\sum_q \bar q_{\mathrm{E}}\left[\frac{1}{2}\gamma_{\mathrm{E}\mu}\overset{\leftrightarrow}{\partial}_\mu+\frac{\mathrm{i}}{2}\gamma_{\mathrm{E}4}(1-r)\overset{\leftrightarrow}{\partial}_3+m_q\right]q_{\mathrm{E}}\\
&+(\partial_4\bar\phi^a_{\mathrm{E}}+\mathrm{i}(1-r)\partial_3\bar\phi^a_{\mathrm{E}})(\partial_4\phi^a_{\mathrm{E}}+\mathrm{i}(1-r)\partial_3\phi^a_{\mathrm{E}})\\
&+(\partial_j\bar\phi^a_{\mathrm{E}})(\partial_j\phi^a_{\mathrm{E}}), \qquad (4.24)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{\mathrm{Int}}_{\mathrm{E},r}&=-gf_{abc}[\partial_4 G^a_{\mathrm{E}j}-\partial_j G^a_{\mathrm{E}4}+\mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}j}]\\
&\times G^b_{\mathrm{E}4}G^c_{\mathrm{E}j}-\frac{1}{2}gf_{abc}(\partial_j G^a_{\mathrm{E}k}-\partial_k G^a_{\mathrm{E}j})G^b_{\mathrm{E}j}G^c_{\mathrm{E}k}\\
&+\frac{1}{4}g^2 f_{abc}f_{ab'c'}G^b_{\mathrm{E}\mu}G^c_{\mathrm{E}\nu}G^{b'}_{\mathrm{E}\mu}G^{c'}_{\mathrm{E}\nu}\\
&+\mathrm{i}g\sum_q \bar q_{\mathrm{E}}\gamma_{\mathrm{E}\mu}G^a_{\mathrm{E}\mu}\frac{1}{2}\lambda_a q_{\mathrm{E}} \qquad (4.25)\\
&-gf_{abc}[(\partial_\mu\bar\phi^a_{\mathrm{E}})G^b_{\mathrm{E}\mu}+\mathrm{i}(1-r)(\partial_3\bar\phi^a_{\mathrm{E}})G^b_{\mathrm{E}4}]\phi^c_{\mathrm{E}}\,.
\end{aligned}
$$

With the same procedure as in [23] we get the following path integral representation for the matrix elements $\widetilde{\mathcal{M}}^-_{a,b}(-\mathrm{i}X_4,r)$ of (4.7) and (4.8):

$$
\begin{aligned}
\widetilde{\mathcal{M}}^-_a(-\mathrm{i}X_4,r)&=\frac{1}{2}\sum_s\lim_{\substack{\tau_i\to-\infty,\\ \tau_f\to+\infty}}\exp[(\tau_f-\tau_i)M]\\
&\times Z_{\mathrm{E}}^{-1}\int\mathcal{D}(G_{\mathrm{E}},q_{\mathrm{E}},\bar q_{\mathrm{E}},\phi_{\mathrm{E}},\bar\phi_{\mathrm{E}})\\
&\times a_{s,\mathrm{E}}(p,\tau_f)(-1)j_{\mathrm{E}\mu}(\mathbf{0},X_4)j_{\mathrm{E}\mu}(0)a_s^\dagger(p,\tau_i)\\
&\times\exp(-S_{\mathrm{E},r}), \qquad (4.26)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{\mathcal{M}}^-_b(-\mathrm{i}X_4,r)&=\frac{1}{2}\sum_s\lim_{\substack{\tau_i\to-\infty,\\ \tau_f\to\infty}}\exp[(\tau_f-\tau_i)M]\\
&\times Z_{\mathrm{E}}^{-1}\int\mathcal{D}(G_{\mathrm{E}},q_{\mathrm{E}},\bar q_{\mathrm{E}},\phi_{\mathrm{E}},\bar\phi_{\mathrm{E}})\\
&\times a_{s,\mathrm{E}}(p,\tau_f)j_{E,4}(\mathbf{0},X_4)j_{\mathrm{E}4}(0)a_s^\dagger(p,\tau_i)\\
&\times\exp(-S_{\mathrm{E},r}). \qquad (4.27)
\end{aligned}
$$

Here

$$Z_{\mathrm{E}}=\int D(G_{\mathrm{E}},q_{\mathrm{E}},\bar q_{\mathrm{E}},\phi_{\mathrm{E}},\bar\phi_{\mathrm{E}})\exp(-S_{\mathrm{E},r}), \qquad (4.28)$$

$$S_{\mathrm{E},r}=\int\mathrm{d}^4X\mathcal{L}_{\mathrm{E},r}(X), \qquad (4.29)$$

$G_{\mathrm{E}},...,\bar\phi_{\mathrm{E}}$ are the integration variables, $j_{\mathrm{E}\mu}$ represent the components of the electromagnetic current and $a_s, a_s^\dagger$ the nucleon state; see Appendix D.

From (4.23)–(4.25) we see that the Euclidean, r-dependent action $S_{\mathrm{E},r}$ has, in general, imaginary parts. To discuss the formal convergence properties of the integrals over the gluon-potential variables we split $S_{\mathrm{E},r}$ into the quadratic and interaction parts

$$S_{\mathrm{E},r}=S^{(0)}_{\mathrm{E},r}+S^{\mathrm{Int}}_{\mathrm{E},r}, \qquad (4.30)$$

$$S^{(0)}_{\mathrm{E},r}=\int\mathrm{d}^4X\mathcal{L}^{(0)}_{\mathrm{E},r}, \qquad (4.31)$$

$$S^{\mathrm{Int}}_{\mathrm{E},r}=\int\mathrm{d}^4X\mathcal{L}^{\mathrm{Int}}_{\mathrm{E},r}. \qquad (4.32)$$

After some partial integrations we get

$$
\begin{aligned}
S^{(0)}_{\mathrm{E},r}&=\int\mathrm{d}^4X\Bigg\{\frac{1}{2}G^a_{\mathrm{E}\mu}\delta_{ab}[-\delta_{\mu\nu}\partial_\lambda\partial_\lambda+(1-\xi^{-1})\partial_\mu\partial_\nu\\
&-2\mathrm{i}(1-r)\delta_{\mu\nu}\partial_3\partial_4\\
&+\mathrm{i}(1-r)(1-\xi^{-1})(\delta_{\mu4}\partial_\nu\partial_3+\delta_{\nu4}\partial_\mu\partial_3)\\
&+(1-r)^2\delta_{\mu\nu}\partial_3\partial_3-(1-r)^2(1-\xi^{-1})\delta_{\mu4}\delta_{\nu4}\partial_3\partial_3]G^b_{\nu E}\\
&+\sum_q\bar q_{\mathrm{E}}[\gamma_{\mathrm{E}\mu}\partial_\mu+\mathrm{i}(1-r)\gamma_{\mathrm{E}4}\partial_3+m_q]q_{\mathrm{E}} \qquad (4.33)\\
&+\bar\phi^a_{\mathrm{E}}\delta_{ab}(-\partial_\lambda\partial_\lambda-2\mathrm{i}(1-r)\partial_3\partial_4+(1-r)^2\partial_3\partial_3)\phi^b_{\mathrm{E}}\Bigg\}.
\end{aligned}
$$

It is particularly convenient to use the Feynman–'t Hooft gauge, that is to set $\xi=1$ in the following. With this we find

$$
\begin{aligned}
S^{(0)}_{\mathrm{E},r}|_{\xi=1}&=\int\mathrm{d}^4X\Bigg\{\frac{1}{2}G^a_{\mathrm{E}\mu}\delta_{ab}\delta_{\mu\nu}\Big[-\partial_\lambda\partial_\lambda-2\mathrm{i}(1-r)\partial_3\partial_4\\
&+(1-r)^2\partial_3\partial_3\Big]G^b_{\mathrm{E}\nu}\\
&+\sum_q\bar q_{\mathrm{E}}[\gamma_{\mathrm{E}\mu}\partial_\mu+\mathrm{i}(1-r)\gamma_{\mathrm{E}4}\partial_3+m_q]q_{\mathrm{E}}\\
&+\bar\phi^a_{\mathrm{E}}\delta_{ab}\Big[-\partial_\lambda\partial_\lambda-2\mathrm{i}(1-r)\partial_3\partial_4\\
&+(1-r)^2\partial_3\partial_3\Big]\phi^b_{\mathrm{E}}\Bigg\}. \qquad (4.34)
\end{aligned}
$$

With the Fourier transformations

$$
\begin{aligned}
G^a_{\mathrm{E}\mu}(X)&=\int\frac{\mathrm{d}^4K}{(2\pi)^4}\mathrm{e}^{\mathrm{i}KX}\widetilde G^a_{\mathrm{E}\mu}(K),\\
(\widetilde G^a_{\mathrm{E}\mu}(K))^*&=\widetilde G^a_{\mathrm{E}\mu}(-K),\\
q_{\mathrm{E}}(X)&=\int\frac{\mathrm{d}^4K}{(2\pi)^4}\mathrm{e}^{\mathrm{i}KX}\tilde q_{\mathrm{E}}(K),\\
\phi^a_{\mathrm{E}}(X)&=\int\frac{\mathrm{d}^4K}{(2\pi)^4}\mathrm{e}^{\mathrm{i}KX}\tilde\phi^a_{\mathrm{E}}(K),\\
\overline{\phi^a_{\mathrm{E}}}(X)&=\int\frac{\mathrm{d}^4K}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}KX}\widetilde{\bar\phi^a}(K), \qquad (4.35)
\end{aligned}
$$

we get

$$
\begin{aligned}
S^{(0)}_{\mathrm{E},r}|_{\xi=1}&=\int\frac{\mathrm{d}^4K}{(2\pi)^4}\Bigg\{\frac{1}{2}(\widetilde G^a_{\mathrm{E}\mu}(K))^*\delta_{ab}\delta_{\mu\nu}\\
&\times[K^2+2\mathrm{i}(1-r)K_3K_4-(1-r)^2K_3^2]\widetilde G^b_{\mathrm{E}\nu}(K)\\
&+\sum_q\tilde{\bar q}_{\mathrm{E}}(K)[\mathrm{i}\gamma_{\mathrm{E}\mu}K_\mu-(1-r)\gamma_{\mathrm{E}4}K_3+m_q]\tilde q_{\mathrm{E}}(K)\\
&+\widetilde{\bar\phi^a}(K)\delta_{ab}\Big[K^2+2\mathrm{i}(1-r)K_3K_4\\
&-(1-r)^2K_3^2\Big]\tilde\phi^b(K)\Bigg\}. \qquad (4.36)
\end{aligned}
$$

Note that

$$K^2 + 2\mathrm{i}(1-r)K_3K_4 - (1-r)^2K_3^2 \qquad (4.37)$$
$$= K_1^2 + K_2^2 + r(2-r)K_3^2 + K_4^2 + 2\mathrm{i}(1-r)K_3K_4$$

has a positive real part, for $0 < r < 2$ and $K \neq 0$.

We discuss now the formal convergence properties of the gluonic integrations in (4.26)–(4.28), setting $\xi = 1$. Consider a given gluon potential $G^a_{\mathrm{E}\mu}(X)$ together with $\lambda G^a_{\mathrm{E}\mu}(X)$, for all $\lambda > 0$, to investigate how the integrand in the functional integral behaves for large potentials. Suppose first that

$$f_{abc}G^b_{\mathrm{E}\mu}(X)G^c_{\mathrm{E}\nu}(X) \not\equiv 0. \qquad (4.38)$$

Then the term in $\mathcal{L}^{\mathrm{Int}}_{\mathrm{E},r}$ (4.25) quartic in the gluon potentials ensures

$$\exp[-S_{\mathrm{E},r}(\lambda G)] \propto \mathrm{e}^{-\lambda^4 c_1}, \quad \text{for} \quad \lambda \to \infty, \qquad (4.39)$$

with $c_1 > 0$. On the other hand, if

$$f_{abc}G^b_{\mathrm{E}\mu}(X)G^c_{\mathrm{E}\nu}(X) = 0, \qquad (4.40)$$

for all X, the positivity of the real part of the quadratic term of the action (4.36) ensures that

$$\exp[-S_{\mathrm{E},r}(\lambda G)|_{\xi=1}] \propto \mathrm{e}^{-\lambda^2 c_2}, \quad \text{for} \quad \lambda \to \infty, \qquad (4.41)$$

with $c_2 > 0$. Thus in any case the integrand of the functional integrals (4.26)–(4.28) is damped exponentially, for large gluon potentials. This should make the integrals well behaved after introducing some regularisation procedure, for instance a lattice regularisation.

4.4 Propagators in Euclidean space and perturbation expansion

In this section we discuss first the lowest-order propagators in Euclidean space for the gauge choice $\xi = 1$. The basic Green's function $\Delta_{\mathrm{E}}(X, m^2, r)$ for mass m in the r-dependent theory is defined through

$$[-\partial_\lambda\partial_\lambda - 2\mathrm{i}(1-r)\partial_3\partial_4 + (1-r)^2\partial_3\partial_3 + m^2]$$
$$\times \Delta_{\mathrm{E}}(X, m^2, r) = \delta^4(X). \qquad (4.42)$$

The solution of (4.42) as given in [23] is

$$\Delta_{\mathrm{E}}(X, m^2, r) = \int \frac{\mathrm{d}^4K}{(2\pi)^4}\mathrm{e}^{\mathrm{i}KX}(K^{\mathrm{T}}A_rK + m^2)^{-1}$$
$$= \frac{m}{4\pi^2}(X^{\mathrm{T}}A_r^{-1}X)^{-1/2}$$
$$\times K_1(m(X^{\mathrm{T}}A_r^{-1}X)^{1/2}), \qquad (4.43)$$

where K_1 is the modified Bessel function of order 1 and

$$A_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r(2-r) & \mathrm{i}(1-r) \\ 0 & 0 & \mathrm{i}(1-r) & 1 \end{pmatrix}, \qquad (4.44)$$

$$A_r^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\mathrm{i}(1-r) \\ 0 & 0 & -\mathrm{i}(1-r) & r(2-r) \end{pmatrix}. \qquad (4.45)$$

For $m^2 = 0$ we get

$$\Delta_{\mathrm{E}}(X, 0, r) = \frac{1}{4\pi^2}(X^{\mathrm{T}}A_r^{-1}X)^{-1}$$
$$= \frac{1}{4\pi^2}(X_1^2 + X_2^2 + X_3^2 + r(2-r)X_4^2$$
$$- 2\mathrm{i}(1-r)X_3X_4)^{-1}. \qquad (4.46)$$

From (4.34) now we easily get the expressions for the propagators in lowest order. The gluon propagator is

$$\Delta^{(0)ab}_{\mathrm{E}\mu\nu}(X, r) = \delta_{ab}\delta_{\mu\nu}\Delta_{\mathrm{E}}(X, 0, r), \qquad (4.47)$$

the quark propagator is

$$S^{(0)}_{\mathrm{E}}(X, m_q, r) = [-\gamma_{\mathrm{E}\mu}\partial_\mu - \mathrm{i}(1-r)\gamma_{\mathrm{E}4}\partial_3 + m_q]$$
$$\times \Delta_{\mathrm{E}}(X, m_q^2, r), \qquad (4.48)$$

and the ghost propagator is

$$\Delta^{(0)ab}_{\mathrm{E}}(X, r) = \delta_{ab}\Delta_{\mathrm{E}}(X, 0, r). \qquad (4.49)$$

All these propagators of (4.43), (4.47)–(4.49) have the property that, for $r \to 0$, they fall off more and more slowly in the X_4 direction, whereas their fall-off in the X_1, X_2 and X_3 directions is independent of r. For the case $m \neq 0$ this is discussed in Sect. 5 of [23]. For the propagator of a quark of mass m_q the correlation length in the directions $X_j (j = 1, 2, 3)$ is

$$l^{(0)}_{\mathrm{T}} = m_q^{-1}, \qquad (4.50)$$

and in the direction X_4:

$$l^{(0)}_4 = [m_q^2 r(2-r)]^{-1/2}$$
$$\approx l^{(0)}_{\mathrm{T}}(2r)^{-1/2}, \quad \text{for } r \to 0. \qquad (4.51)$$

For the massless case (4.46) there is of course no genuine correlation length, but for $r \to 0$, the fall-off in the X_4 direction is again slower by a factor $(2r)^{-1/2}$ compared to the $X_{1,2,3}$ directions.

We can now use the propagators (4.47)–(4.49) to construct the unrenormalised perturbation series for the Green's functions in the theory described by $\mathcal{L}_{\mathrm{E},r}$ (4.22). Since the Fourier transforms of the propagators (4.43), (4.47)–(4.49) have no unwanted poles in momentum space, the convergence properties of Feynman integrals should be the same, for arbitrary r, as for the standard case $r = 1$. Thus we conclude that also the construction of the renormalised perturbation series should work for the $\mathcal{L}_{\mathrm{E},r}$ theory as for the standard case. Of course, the purpose of our paper is not to advocate perturbative calculations starting with the Lagrangian density $\mathcal{L}_{\mathrm{E},r}$ of (4.23) but to provide a framework for non-perturbative calculations.

5 Phenomenological applications

In this section we study the naive parton model from our point of view and make some speculations concerning a possible critical behaviour of the full theory with interaction, for $r \to 0$.

5.1 The naive parton model

The central assumption of the naive parton model is that of free field or canonical behaviour of the product of currents near the lightcone. In detail one assumes [29] for $\mathcal{M}_{1,2}$ of (3.5)

$$\begin{aligned}\mathcal{M}_1(xp,x^2) &\sim h_1(xp)\frac{4\pi^2}{\mathrm{i}}\Delta^+(x,m^2)\\ &\sim h_1(xp)(-x^2+\mathrm{i}\epsilon(xp))^{-1}, \qquad (5.1)\\ \mathcal{M}_2(xp,x^2) &\sim -h_2(xp)8\pi^2\int_0^\infty \mathrm{d}s\delta'(s-m^2)\frac{1}{\mathrm{i}}\Delta^+(x,s)\\ &\sim \frac{1}{2}h_2(xp)\ln[m^2(-x^2+\mathrm{i}\epsilon(xp))], \qquad (5.2)\end{aligned}$$

where $\sim$ indicates that only the leading term for $x^2 \to 0$ is considered and Δ^+ is defined in (3.20). The parameter m represents a hadronic mass scale. Its precise value does not matter except that we will assume

$$0 < m \le M. \qquad (5.3)$$

According to the DGS representations (3.22) and (3.23) the functions $h_{1,2}(xp)$ can be represented as

$$h_1(xp) = \int_{-1}^{1}\mathrm{d}\zeta[\cos(\zeta px)-1]\tilde{h}_1(\zeta)+h_1^{(0)}, \qquad (5.4)$$

$$h_2(xp) = \int_{-1}^{1}\mathrm{d}\zeta\cos(\zeta px)\tilde{h}_2(\zeta), \qquad (5.5)$$

where

$$\begin{aligned}\tilde{h}_j(\zeta) &= \tilde{h}_j(-\zeta) = \tilde{h}_j^*(\zeta) \quad (j=1,2),\\ h_1^{(0)} &= h_1^{(0)*} = \text{const}. \qquad (5.6)\end{aligned}$$

It is now an easy exercise to calculate the structure functions $W_{1,2}$ inserting the parton model ansatz (5.1)–(5.6) into (3.8) and (3.9). The result, expressed in terms of $F_{2,\mathrm{L}}$ of (2.8), is

$$F_2(x_{\rm Bj},Q^2) = \nu W_2(\nu,Q^2) \sim 2\pi^2 x_{\rm Bj}\tilde{h}'_2(x_{\rm Bj}), \qquad (5.7)$$

$$F_{\rm L}(x_{\rm Bj},Q^2) \sim 4\pi^2 x_{\rm Bj}^2\tilde{h}_1(x_{\rm Bj}), \qquad (5.8)$$

where now $\sim$ indicates the leading term, for $Q^2 \to \infty$. Of course, we get Bjorken scaling. From the well-known relations of the parton model we get the physical interpretation of the functions $\tilde{h}_1$ and $\tilde{h}_2$ as

$$\begin{aligned}2\pi^2\tilde{h}'_2(\zeta) &= \sum_j e_j^2 N_j(\zeta) + \sum_j \tilde{e}_j^2\tilde{N}_j(\zeta),\\ 4\pi^2\tilde{h}_1(\zeta) &= \zeta^{-1}\sum_j \tilde{e}_j^2\tilde{N}_j(\zeta),\\ &0<\zeta\le 1, \qquad (5.9)\end{aligned}$$

where $e_j, N_j(\zeta)$ are the charges and distribution functions for the spin 1/2 partons and $\tilde{e}_j, \tilde{N}_j(\zeta)$ for the spin 0 partons.

Using an approximate parton model description, we conclude from experiment [1–3] and the fits in [22] that the growth of F_2, for $x_{\rm Bj}\to 0$, is at most as

$$F_2(x_{\rm Bj},Q^2) \propto (x_{\rm Bj})^{-a}, \qquad (5.10)$$

with $0<a<1$. This implies from (5.7) that at most

$$\begin{aligned}\tilde{h}'_2(\zeta) &\propto |\zeta|^{-a-1},\\ \tilde{h}_2(\zeta) &\propto |\zeta|^{-a}, \qquad (5.11)\end{aligned}$$

for $\zeta\to 0$. The structure function $F_{\rm L}$ is not well measured at high Q^2. All indications are that $F_{\rm L}$ is small compared to F_2. Thus a conservative estimate is that also $F_{\rm L}$ grows at most as

$$F_{\rm L}(x_{\rm Bj},Q^2) \propto (x_{\rm Bj})^{-a}, \qquad (5.12)$$

for $x_{\rm Bj}\to 0$. From (5.8) this implies that at most

$$\tilde{h}_1(\zeta) \propto |\zeta|^{-a-2}, \qquad (5.13)$$

for $\zeta \to 0$. We see that with (5.11) and (5.13) the integrals (5.5) and (5.4) are perfectly convergent, for $\zeta = 0$. On the other hand, an unsubtracted integral as for h_2 would not be convergent, for h_1. These findings are our motivation to write the DGS representations (3.22) without, and (3.23) with subtractions.

The purpose of our discussion of the parton model is to see how one obtains in this case a relation between the behaviour of the structure functions at large ν, that is small $x_{\rm Bj}$, of the amplitudes $T_{1,2}$ of (3.35) and (3.36) at large η, that is large imaginary ν, and of the matrix elements $\widetilde{\mathcal{M}}_{1,2}$ (3.28) and $\widetilde{\mathcal{M}}^-_{a,b}$ (4.1), (4.2) at small r. For illustration we consider only W_2, T_2 and $\widetilde{\mathcal{M}}_2$ and make the following simple ansatz:

$$2\pi^2\tilde{h}_2(\zeta) = -A\frac{1}{a}(|\zeta|^{-a}-1)(1-|\zeta|)^b, \qquad (5.14)$$

with

$$0<a<1, \quad b>1, \quad A>0. \qquad (5.15)$$

We get then, for $0<\zeta\le 1$,

$$\begin{aligned}2\pi^2\tilde{h}'_2(\zeta) &= A\left[1-\zeta+\frac{b}{a}\zeta(1-\zeta^a)\right]\zeta^{-a-1}(1-\zeta)^{b-1}\\ &\sim A\zeta^{-a-1}, \quad \text{for small } \zeta \qquad (5.16)\end{aligned}$$

and from (5.7), for small $x_{\rm Bj}$,

$$F_2(x_{\rm Bj},Q^2) \sim A(x_{\rm Bj})^{-a}, \qquad (5.17)$$

which implies, for large ν at fixed Q^2,

$$W_2(\nu,Q^2) \sim \frac{A}{\nu}\left(\frac{2M\nu}{Q^2}\right)^a, \qquad (5.18)$$

$$\sigma_{\rm T}(\nu,Q^2)+\sigma_{\rm L}(\nu,Q^2) \sim 4\pi^2\alpha\frac{A}{Q^2}\left(\frac{2M\nu}{Q^2}\right)^a. \qquad (5.19)$$

On the other hand, we have argued at the end of Sect. 3 that the behaviour of $T_2(\mathrm{i}\eta,Q^2)$, for $\eta\to\infty$, should be governed by the behaviour of $\widetilde{\mathcal{M}}_2(x^0,r)$ of (3.28), for small $|r|$ and large x^0. With the ansatz (5.2) we get

$$\widetilde{\mathcal{M}}_2(x^0,r) \sim \frac{1}{2} h_2(x^0M)\ln(-r), \qquad (5.20)$$

for small $|r|$, where from (5.5) and (5.14):

$$h_2(x^0M) = -\frac{A}{\pi^2 a}\int_0^1 \mathrm{d}\zeta \cos(\zeta x^0 M)(\zeta^{-a}-1)(1-\zeta)^b. \qquad (5.21)$$

To see the large x^0 behaviour of $h_2(x^0M)$, we write it as follows:

$$\begin{aligned} h_2(x^0M) = &-\frac{A}{\pi^2 a}\int_0^\infty \mathrm{d}\zeta \cos(\zeta x^0M)\zeta^{-a}\mathrm{e}^{-\zeta} \\ &-\frac{A}{\pi^2 a}\int_0^\infty \mathrm{d}\zeta \cos(\zeta x^0 M) \qquad (5.22)\\ &\times [(\zeta^{-a}-1)(1-\zeta)^b\theta(1-\zeta)-\zeta^{-a}e^{-\zeta}]. \end{aligned}$$

The second integral on the r.h.s. of (5.22) vanishes faster than $1/x^0$, for $x^0\to\infty$, as we find from a simple application of the Riemann–Lebesgue lemma. The first integral then gives the leading behaviour, for large x^0,

$$h_2(x^0M) \sim -\frac{A}{\pi^2 a}\Gamma(1-a)\sin\left(\frac{\pi}{2}a\right)(x^0M)^{a-1}. \qquad (5.23)$$

Inserting (5.20) and (5.23) into (3.36) gives, for large η,

$$\begin{aligned} T_2(\mathrm{i}\eta,Q^2) &\sim -\frac{\mathrm{i}MQ^2}{2\eta}\frac{AM^{a-1}}{\pi^2 a}\Gamma(1-a)\sin\left(\frac{\pi}{2}a\right) \\ &\times \int_C \mathrm{d}r \ln(-r)\int_0^\infty \mathrm{d}x^0 (x^0)^{1+a}\exp(-x^0/\bar{x}(\eta,r)) \\ &\sim -\frac{\mathrm{i}MQ^2}{2\eta^{3+a}}\frac{A}{\pi^2 a}M^{a-1}\Gamma(2+a)\Gamma(1-a)\sin\left(\frac{\pi}{2}a\right) \\ &\times \int_C \mathrm{d}r\ln(-r)(r+\bar{r})^{-2-a} \\ &\sim A\left(\cos\left(\frac{\pi}{2}a\right)\right)^{-1}\frac{1}{\eta}\left(\frac{Q^2}{2\eta M}\right)^{-a}. \qquad (5.24) \end{aligned}$$

With analytic continuation we find, for large $|\nu|$,

$$\begin{aligned} T_2(\nu,Q^2) &\sim A\left(\cos\left(\frac{\pi}{2}a\right)\right)^{-1} \qquad (5.25)\\ &\times \left[\sin\left(\frac{\pi}{2}a\right)+\mathrm{i}\cos\left(\frac{\pi}{2}a\right)\right]\frac{1}{\nu}\left(\frac{Q^2}{2M\nu}\right)^{-a} \end{aligned}$$

and, for large real ν,

$$W_2(\nu,Q^2) = \mathrm{Im}T_2(\nu+\mathrm{i}\epsilon,Q^2) \sim A\frac{1}{\nu}\left(\frac{2M\nu}{Q^2}\right)^a. \qquad (5.26)$$

Of course, this is in perfect agreement with (5.18).

We have thus seen in this simple example that the power behaviour (5.17) of F_2 corresponds to a behaviour

$$\widetilde{\mathcal{M}}_2(x^0,r) \propto (x^0M)^{a-1}\ln(-r), \qquad (5.27)$$

for small $|r|$ and large x^0 (see (5.20) and (5.23)). If we assume that $\widetilde{\mathcal{M}}_1(x^0,r)$ has a similar behaviour, as is for instance true for vanishing $\sigma_{\rm L}$ (see (3.11)), we get from (5.27) and (A.1), (A.2) for the current correlation functions (4.7) and (4.8):

$$\widetilde{\mathcal{M}}^-_{a,b}(-\mathrm{i}X_4,r) \propto (-\mathrm{i}X_4M)^{a-3}(-r)^{-2}, \qquad (5.28)$$

for small $|r|$ and large X_4.

5.2 Regge behaviour

We have seen in Sect. 4.4 that the free propagators in the r-dependent Euclidean theory develop in the X_4 direction a large correlation length $\propto 1/r^{1/2}$, for $r\to 0$. Let us assume here that this property remains true in the full theory with interaction. In [23] it has been argued that one could then expect to see a critical behaviour of the theory, for $r\to 0$, with r playing the role of the deviation of the temperature from the critical one, in the statistical physics of a system near a second-order phase transition at $T_{\rm c}$,

$$r \sim (T-T_{\rm c})/T_{\rm c}. \qquad (5.29)$$

We can then expect to see a simple power behaviour of the correlation functions in the X_4 direction, for $r\to 0$, from general scaling and renormalisation group arguments. In [23] various possibilities for the behaviour of the correlation length in the X_4 direction are discussed.

Let us assume here, as an example, that the r-dependent theory of Sect. 4.3 has a correlation length in the X_4 direction similar to (4.51):

$$l_4(r) \sim m^{-1}(-r)^{-1/2}, \qquad (5.30)$$

for $r\to 0$, where m is a hadronic mass scale. We assume, furthermore, a simple power behaviour of the correlation functions (4.7) and (4.8):

$$\begin{aligned} &\widetilde{\mathcal{M}}^-_{a,b}(-\mathrm{i}X_4,r) \propto (-\mathrm{i}X_4M)^{a-3}(-r)^{-2-(1/2)\varepsilon_0}, \\ &0\le a<1, \quad |\varepsilon_0|<1, \qquad (5.31) \end{aligned}$$

for $r\to 0$ and $m^{-1} \ll X_4 \ll |l_4(r)|$. The ansatz (5.31) corresponds to (see (A.1) and (A.2))

$$\begin{aligned} \widetilde{\mathcal{M}}_2(x^0,r) &\sim A'(x^0M)^{a-1}(-r)^{-(1/2)\varepsilon_0}, \\ A' &= \text{const.}, \qquad (5.32) \end{aligned}$$

for $r\to 0$ and $m^{-1} \ll x^0 \ll |l_4(r)|$. What are the consequences of (5.32) for $T_2(\mathrm{i}\eta,Q^2)$ of (3.36) and the structure function $F_2(x_{\rm Bj},Q^2)$ of (2.8)? We have argued at the end of Sect. 3.3 that the relevant region of the r integration in (3.36) is for $r\approx -\bar{r}/2$. Consider now the x^0 integral in (3.36) for this value of r. We have on the one hand the exponential damping factor $\exp(-x^0/\bar{x}(\eta,r))$; on the other hand, the matrix element $\widetilde{\mathcal{M}}_2(x^0,r)$ provides as cut-off the correlation length $l_4(r)$. For $r = -\bar{r}/2$ we get from (3.34) and (5.30), for $\eta \gg Q$,

$$\begin{aligned} \bar{r} &\approx \frac{Q^2}{2\eta^2}, \\ \bar{x}\left(\eta, -\frac{1}{2}\bar{r}\right) &\approx \frac{4\eta}{Q^2}, \\ l_4\left(-\frac{1}{2}\bar{r}\right) &\approx \frac{2\eta}{mQ}. \end{aligned} \tag{5.33}$$

This implies

$$\begin{aligned} \bar{x}\left(\eta, -\frac{1}{2}\bar{r}\right) &> l_4\left(-\frac{1}{2}\bar{r}\right), \quad \text{for } Q < 2m, \\ \bar{x}\left(\eta, -\frac{1}{2}\bar{r}\right) &< l_4\left(-\frac{1}{2}\bar{r}\right), \quad \text{for } Q > 2m. \end{aligned} \tag{5.34}$$

Thus, for $\eta \gg Q \gg m$, the x^0 integration in (3.36) is effectively cut off at $x^0 = \bar{x}(\eta, -\bar{r}/2)$ and the x^0 integral in (3.36) will get its leading contribution from the scaling regime of $\widetilde{\mathcal{M}}_2(x^0, r)$; see (5.32). Inserting (5.32) in (3.36) we get, for $\eta \gg Q \gg m$,

$$\begin{aligned} T_2(\mathrm{i}\eta, Q^2) &\sim \frac{\mathrm{i}MQ^2}{\eta}\int_C \mathrm{d}r \int_0^\infty \mathrm{d}x^0 (x^0)^2 \\ &\quad \times \exp(-x^0/\bar{x}(\eta, r)) A'(x^0 M)^{a-1}(-r)^{-(1/2)\varepsilon_0} \\ &\sim \mathrm{i}\frac{Q^2}{M^3}\left(\frac{M}{\eta}\right)^{3+a} \Gamma(2+a)A' \\ &\quad \times \int_C \mathrm{d}r (-r)^{-(1/2)\varepsilon_0}(r+\bar{r})^{-2-a} \\ &\sim \frac{4\pi}{M}A' \frac{\Gamma\left(1+a+\frac{1}{2}\varepsilon_0\right)}{\Gamma\left(\frac{1}{2}\varepsilon_0\right)} \\ &\quad \times \left(\frac{Q^2}{2M^2}\right)^{-a-(1/2)\varepsilon_0} \left(\frac{\eta}{M}\right)^{-1+a+\varepsilon_0}. \end{aligned} \tag{5.35}$$

The analytic continuation, for arbitrary large $|\nu|$, is obtained by the replacement in (5.35):

$$\eta \to \nu \exp\left(-\mathrm{i}\frac{1}{2}\pi\right). \tag{5.36}$$

For real positive ν this leads to

$$\begin{aligned} \nu W_2(\nu, Q^2) &= F_2(x_{\rm Bj}, Q^2) \\ &\sim 4\pi A' \Gamma\left(1+a+\frac{1}{2}\varepsilon_0\right)\left(\Gamma\left(\frac{1}{2}\varepsilon_0\right)\right)^{-1} \\ &\quad \times \sin\left[\frac{\pi}{2}(1-a-\varepsilon_0)\right]\left(\frac{Q^2}{2M^2}\right)^{(1/2)\varepsilon_0} (x_{\rm Bj})^{-a-\varepsilon_0}. \end{aligned} \tag{5.37}$$

This is an interesting result. The simple scaling assumption for the matrix element $\widetilde{\mathcal{M}}_2$ (5.32) as suggested by the analogy to critical phenomena leads to the behaviour for the structure function F_2 found in the Regge fit (see (2) and (4a) of [22]), for large Q^2 and small $x_{\rm Bj}$,

$$F_2(x_{\rm Bj}, Q^2) \sim X_0 (Q_0^2)^{1+\varepsilon_0}\left(\frac{Q^2}{Q_0^2}\right)^{(1/2)\varepsilon_0} (x_{\rm Bj})^{-\varepsilon_0}. \tag{5.38}$$

Here $X_0, Q_0^2, \varepsilon_0$ are constants with

$$\varepsilon_0 \approx 0.44, \tag{5.39}$$

where $1+\varepsilon_0$ is the intercept of the hard pomeron. Setting $a = 0$ in (5.37) we get indeed the powers of Q^2 and $x_{\rm Bj}$ of (5.38). Comparing (5.28) and (5.31) we see that $(1/2)\varepsilon_0$ can be interpreted as the anomalous part of a critical index in the language of statistical physics.

6 Conclusions

In this article we have developed an approach for the theoretical description of the structure functions at small $x_{\rm Bj}$ which should allow truly non-perturbative calculations for all $Q^2 > 0$ to be made. Of course, one can also perform perturbative calculations in this framework. We have introduced an effective, r-dependent theory (see Sect. 4) in Minkowski and Euclidean space, starting from QCD in Feynman–'t Hooft gauge. We have argued that the small $x_{\rm Bj}$ behaviour of the structure functions is related to the small r behaviour of the effective theory and that the limit $r \to 0$ corresponds to a critical point. In the vicinity of this critical point we can expect to see power behaviour of the relevant matrix elements $\widetilde{\mathcal{M}}_{1,2}(x^0, r)$ (3.28) with certain critical indices. We have shown that this leads to Regge behaviour of the structure functions at small $x_{\rm Bj}$ and large Q^2, as observed in the phenomenological fits of [22]. In this way the intercept of the hard pomeron is related to a critical index of the r-dependent theory. We emphasise that in principle both the hard and the soft pomeron contributions to the structure functions should be calculable in our approach.

The idea that the small $x_{\rm Bj}$ behaviour of the structure functions may be related to some kind of critical behaviour has been suggested by various authors. Our present article follows the ideas presented in [23]. The critical point of our effective theory for $r \to 0$, if it is indeed confirmed, would be completely analogous to a point of a second-order phase transition in statistical physics. Quite a different type of critical behaviour, self-organised criticality, was suggested for the small $x_{\rm Bj}$ behaviour of the structure functions in [31]. Criticality of the photon wave function in connection with a dipole model was suggested in [15]. It is also interesting to note that perturbative calculations in the leading logarithmic approximation, that is in the framework of the BFKL equation [10], lead to conformal invariant structures for the amplitudes [32, 33]. One can suspect that this conformal invariance could have its origin in some sort of critical behaviour of an effective theory.

Coming back to our present article we have presented here also our results on some more technical issues.

We have discussed various ways of analytic continuation of the Compton amplitude in the ν- and r-planes. We found the representations (3.29)–(3.36) the most useful ones. The r-dependent theory in Euclidean space was studied in particular in the Feynman–'t Hooft gauge. The final goal of our approach is to make a truly non-perturbative calculation of the matrix elements (4.26) and (4.27) in the r-dependent Euclidean theory. This could be based for instance on exact renormalisation group methods (see [34, 35] for a review) or on lattice methods. Certainly, it will not be an easy task, since our Euclidean action (4.29) has an imaginary part, for $r \neq 1$. But suppose that one can indeed study in this way the theory first for real r with $0 < r < 2$, deduce critical behaviour, for small r, and establish scaling relations as in (5.31). Making then the analytic continuation in r and putting everything into the theoretical machinery developed in this paper would mean a calculation of the small $x_{\rm Bj}$ and large Q^2 behaviour of the structure functions. Both the functional dependences and the absolute normalisation of the structure functions could be obtained in this way. Of course, much remains to be done.

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Appendix

A Relation of $\widetilde{\mathcal{M}}^{-}_{a,b}$ and $\widetilde{\mathcal{M}}^{-}_{1,2}$

From the definitions (3.16) and (4.1), (4.2) and using (3.5) we get after some straightforward but lengthy algebra:

$$\begin{aligned}\widetilde{\mathcal{M}}^{-}_{a}(x^0,r) &= \left\{-3\frac{\partial^2}{(\partial x^0)^2} - \frac{6(1-r)}{x^0}\frac{\partial^2}{\partial x^0 \partial r}\right.\\ &\left. + \frac{3r(2-r)}{(x^0)^2}\frac{\partial^2}{(\partial r)^2} - \frac{6r(2-r)}{(x^0)^2(1-r)}\frac{\partial}{\partial r}\right\}\widetilde{\mathcal{M}}^{-}_{1}(x^0,r)\\ &+\left\{-3M^2\frac{\partial^2}{(\partial x^0)^2} - \frac{6(1-r)M^2}{x^0}\frac{\partial^2}{\partial x^0\partial r}\right.\\ &+\frac{3r(2-r)-2}{(x^0)^2}M^2\frac{\partial^2}{(\partial r)^2}\\ &\left.-\frac{3r(2-r)-2}{(x^0)^2(1-r)}2M^2\frac{\partial}{\partial r}\right\}\widetilde{\mathcal{M}}^{-}_{2}(x^0,r), \qquad \text{(A.1)}\end{aligned}$$

$$\begin{aligned}\widetilde{\mathcal{M}}^{-}_{b}(x^0,r) &= -(x^0)^{-2}\left\{\frac{\partial^2}{\partial r^2} - \frac{2}{1-r}\frac{\partial}{\partial r}\right\}\\ &\times\{\widetilde{\mathcal{M}}^{-}_{1}(x^0,r) + M^2\widetilde{\mathcal{M}}^{-}_{2}(x^0,r)\}. \qquad \text{(A.2)}\end{aligned}$$

B Effective Lagrangian density in covariant gauges

Here we give the details of the derivation of $\mathcal{L}_r(x)$ of (4.14). From the original Lagrangian $\mathcal{L}$ (4.9) we get the canonical momenta in the standard way. It is convenient to write first $\mathcal{L}$ separating time and space components of the fields. We have

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\dot{G}^{aj} + \partial_j G^{a0} - g f_{abc} G^{b0} G^{cj})\\ &\times (\dot{G}^{aj} + \partial_j G^{a0} - g f_{ab'c'} G^{b'0} G^{c'j})\\ &- \frac{1}{4}G^a_{jk}G^{ajk} - \frac{1}{2\xi}(\dot{G}^{a0} + \partial_j G^{aj})(\dot{G}^{a0} + \partial_k G^{ak})\\ &+ \sum_q \left\{\bar{q}\frac{\mathrm{i}}{2}\gamma^0\dot{q} - \dot{\bar{q}}\frac{\mathrm{i}}{2}\gamma^0 q\right.\\ &\left. + \bar{q}\left(\frac{\mathrm{i}}{2}\gamma^j \overleftrightarrow{\partial}_j - g\gamma^\mu G^a_\mu \frac{\lambda_a}{2} - m_q\right) q\right\}\\ &+ \dot{\bar{\phi}}^a\dot{\phi}^a - (\partial_j\bar{\phi}^a)\partial_j\phi^a\\ &- g f_{abc}\{\dot{\bar{\phi}}^a G^{b0}\phi^c + (\partial_j\bar{\phi}^a)G^{bj}\phi^c\}. \qquad \text{(B.1)}\end{aligned}$$

We now easily get

$$\Pi_{G^{a0}} = \frac{\partial\mathcal{L}}{\partial\dot{G}^{a0}} = -\frac{1}{\xi}(\dot{G}^{a0} + \partial_j G^{aj}), \qquad \text{(B.2)}$$

$$\Pi_{G^{aj}} = \frac{\partial\mathcal{L}}{\partial\dot{G}^{aj}} = \dot{G}^{aj} + \partial_j G^{a0} - g f_{abc}G^{b0}G^{cj}, \qquad \text{(B.3)}$$

$$\begin{aligned}\Pi_q &= \frac{\partial\mathcal{L}}{\partial\dot{q}} = \bar{q}\frac{\mathrm{i}}{2}\gamma^0,\\ \Pi_{\bar{q}} &= \frac{\partial\mathcal{L}}{\partial\dot{\bar{q}}} = -\frac{\mathrm{i}}{2}\gamma^0 q, \qquad \text{(B.4)}\end{aligned}$$

$$\begin{aligned}\Pi_{\phi^a} &= \frac{\partial\mathcal{L}}{\partial\dot{\phi}^a} = \dot{\bar{\phi}}^a,\\ \Pi_{\bar{\phi}^a} &= \frac{\partial\mathcal{L}}{\partial\dot{\bar{\phi}}^a} = \dot{\phi}^a - g f_{abc}G^{b0}\phi^c. \qquad \text{(B.5)}\end{aligned}$$

Solving (B.2), (B.3) and (B.5) for the time derivatives of the fields, we get

$$\begin{aligned}\dot{G}^{a0} &= -\xi\Pi_{G^{a0}} - \partial_j G^{aj},\\ \dot{G}^{aj} &= \Pi_{G^{aj}} - \partial_j G^{a0} + g f_{abc}G^{b0}G^{cj},\\ \dot{\bar{\phi}}^a &= \Pi_{\phi^a},\\ \dot{\phi}^a &= \Pi_{\bar{\phi}^a} + g f_{abc}G^{b0}\phi^c. \qquad \text{(B.6)}\end{aligned}$$

The Hamiltonian density is

$$\begin{aligned}\mathcal{H} &= \Pi_{G^{a0}}\dot{G}^{a0} + \Pi_{G^{aj}}\dot{G}^{aj} + \sum_q(\Pi_q\dot{q} + \dot{\bar{q}}\Pi_{\bar{q}})\\ &+ \Pi_{\phi^a}\dot{\phi}^a + \dot{\bar{\phi}}^a\Pi_{\bar{\phi}^a} - \mathcal{L}, \qquad \text{(B.7)}\end{aligned}$$

or written out explicitly:

$$\begin{aligned}\mathcal{H} &= -\frac{1}{2}\xi\Pi_{G^{a0}}\Pi_{G^{a0}} - \Pi_{G^{a0}}\partial_j G^{aj}\\ &+ \frac{1}{2}\Pi_{G^{aj}}\Pi_{G^{aj}} + \frac{1}{4}G^a_{jk}G^{ajk}\\ &+ \Pi_{G^{aj}}(-\partial_j G^{a0} + g f_{abc}G^{b0}G^{cj})\\ &+ \sum_q\left\{\bar{q}\left(-\frac{\mathrm{i}}{2}\gamma^j\overleftrightarrow{\partial}_j + g\gamma^\mu G^a_\mu\frac{\lambda_a}{2} + m_q\right)q\right\}\end{aligned}$$

$$+ \Pi_{\phi^a}\Pi_{\bar{\phi}^a} + \Pi_{\phi^a} g f_{abc} G^{b0}\phi^c + (\partial_j\bar{\phi}^a)\partial_j\phi^a + g f_{abc}(\partial_j\bar{\phi}^a)G^{bj}\phi^c. \quad \text{(B.8)}$$

The 03 component of the canonical energy-momentum tensor is

$$\mathcal{T}_{03} = \Pi_{G^{a0}}\partial_3 G^{a0} + \Pi_{G^{aj}}\partial_3 G^{aj} + \Pi_q\partial_3 q + (\partial_3\bar{q})\Pi_{\bar{q}} + \Pi_{\phi^a}\partial_3\phi^a + (\partial_3\bar{\phi}^a)\Pi_{\bar{\phi}^a}. \quad \text{(B.9)}$$

We have, furthermore,

$$P^3 = \int \mathrm{d}^3x\mathcal{T}^{03} = -\int \mathrm{d}^3x\mathcal{T}_{03} \quad \text{(B.10)}$$

and from the definitions (4.4) and (4.12)

$$H_r = P^0 - (1-r)P^3 = \int \mathrm{d}^3x\{\mathcal{H} + (1-r)\mathcal{T}_{03}\} = \int \mathrm{d}^3x\mathcal{H}_r. \quad \text{(B.11)}$$

Thus, we get

$$\mathcal{H}_r = \mathcal{H} + (1-r)\mathcal{T}_{03}. \quad \text{(B.12)}$$

Inserting here (B.8) and (B.9) leads to (4.13).

To prove that $\mathcal{H}_r$ of (4.13) is the Hamiltonian density to the Lagrangian $\mathcal{L}_r(x)$ of (4.14) we proceed again in the standard way. We first write down $\mathcal{L}_r(x)$ separating time and space components of the fields:

$$\begin{aligned}
\mathcal{L}_r =& \frac{1}{2}(\dot{G}^{aj} + \partial_j G^{a0} - g f_{abc}G^{b0}G^{cj} - (1-r)\partial_3 G^{aj}) \\
&\times (\dot{G}^{aj} + \partial_j G^{a0} - g f_{ab'c'}G^{b'0}G^{c'j} - (1-r)\partial_3 G^{aj}) \\
&- \frac{1}{4}G^a_{jk}G^{ajk} \\
&- \frac{1}{2\xi}(\dot{G}^{a0} + \partial_j G^{aj} - (1-r)\partial_3 G^{a0}) \\
&\times (\dot{G}^{a0} + \partial_k G^{ak} - (1-r)\partial_3 G^{a0}) \\
&+ \sum_q \left\{\bar{q}\frac{\mathrm{i}}{2}\gamma^0\dot{q} - \dot{\bar{q}}\frac{\mathrm{i}}{2}\gamma^0 q \right.\\
&\left.+ \bar{q}\left(\frac{\mathrm{i}}{2}\gamma^j \overleftrightarrow{\partial}_j - (1-r)\frac{\mathrm{i}}{2}\gamma^0 \overleftrightarrow{\partial}_3 - g\gamma^\mu G^a_\mu\frac{\lambda_a}{2} - m_q\right)q\right\} \\
&+ (\dot{\bar{\phi}}^a - (1-r)\partial_3\bar{\phi}^a)\Big(\dot{\phi}^a - g f_{abc}G^{b0}\phi^c \\
&- (1-r)\partial_3\phi^a\Big) \\
&- (\partial_j\bar{\phi}^a)\partial_j\phi^a - g f_{abc}(\partial_j\bar{\phi}^a)G^{bj}\phi^c. \qquad \text{(B.13)}
\end{aligned}$$

From (B.13) we find the new canonical momenta to be

$$\begin{aligned}
\Pi_{G^{a0}} &= \frac{\partial\mathcal{L}_r}{\partial\dot{G}^{a0}} \\
&= -\frac{1}{\xi}(\dot{G}^{a0} + \partial_j G^{aj} - (1-r)\partial_3 G^{a0}), \qquad \text{(B.14)} \\
\Pi_{G^{aj}} &= \frac{\partial\mathcal{L}_r}{\partial\dot{G}^{aj}} \\
&= \dot{G}^{aj} + \partial_j G^{a0} - g f_{abc}G^{b0}G^{cj} \\
&- (1-r)\partial_3 G^{aj}, \qquad \text{(B.15)} \\
\Pi_{\phi^a} &= \frac{\partial\mathcal{L}_r}{\partial\dot{\phi}^a} = \dot{\bar{\phi}}^a - (1-r)\partial_3\bar{\phi}^a, \\
\Pi_{\bar{\phi}^a} &= \frac{\partial\mathcal{L}_r}{\partial\dot{\bar{\phi}}^a} = \dot{\phi}^a - g f_{abc}G^{b0}\phi^c - (1-r)\partial_3\phi^a \qquad \text{(B.16)}
\end{aligned}$$

and Π_q, $\Pi_{\bar{q}}$ staying as in (B.4).

Solving (B.14), (B.15) and (B.16) for the time derivatives of the fields, we get

$$\begin{aligned}
\dot{G}^{a0} &= -\xi\Pi_{G^{a0}} - \partial_j G^{aj} + (1-r)\partial_3 G^{a0}, \\
\dot{G}^{aj} &= \Pi_{G^{aj}} - \partial_j G^{a0} + g f_{abc}G^{b0}G^{cj} \\
&+ (1-r)\partial_3 G^{aj}, \qquad \text{(B.17)} \\
\dot{\bar{\phi}}^a &= \Pi_{\phi^a} + (1-r)\partial_3\bar{\phi}^a, \\
\dot{\phi}^a &= \Pi_{\bar{\phi}^a} + g f_{abc}G^{b0}\phi^c + (1-r)\partial_3\phi^a. \qquad \text{(B.18)}
\end{aligned}$$

With this we find that $\mathcal{H}_r$ of (4.13) is precisely given as

$$\mathcal{H}_r = \Pi_{G^{a0}}\dot{G}^{a0} + \Pi_{G^{aj}}\dot{G}^{aj} + \sum_q(\Pi_q\dot{q} + \dot{\bar{q}}\Pi_{\bar{q}}) + \Pi_{\phi_a}\dot{\phi}^a + \dot{\bar{\phi}}^a\Pi_{\bar{\phi}^a} - \mathcal{L}_r. \quad \text{(B.19)}$$

C Effective Lagrangian density in the temporal gauge

Here we give the r-dependent Lagrangian and Hamiltonian densities in the temporal gauge. We start from the standard Lagrangian of QCD and add a total divergence term (see Appendix E of [25]). We can write

$$\begin{aligned}
\tilde{\mathcal{L}} =& -\frac{1}{4}G^a_{\lambda\rho}G^{a\lambda\rho} + \sum_q \bar{q}(\mathrm{i}\gamma^\lambda D_\lambda - m_q)q \\
&+ \partial_\lambda\left\{\frac{1}{2}(G^{a\lambda}\partial_\rho G^{a\rho} - G^{a\rho}\partial_\rho G^{a\lambda}) - \sum_q \bar{q}\frac{\mathrm{i}}{2}\gamma^\lambda q\right\}, \\
D_\lambda =& \partial_\lambda + \mathrm{i}gG^a_\lambda\frac{1}{2}\lambda_a. \qquad \text{(C.1)}
\end{aligned}$$

With the gauge condition

$$G^a_0(x) = 0 \quad (a = 1,\ldots,8), \qquad \text{(C.2)}$$

we get

$$\begin{aligned}
\tilde{\mathcal{L}} =& \frac{1}{2}\dot{G}^{aj}\dot{G}^{aj} - \frac{1}{2}(\partial_j G^{ak})\partial_j G^{ak} + \frac{1}{2}(\partial_j G^{aj})\partial_k G^{ak} \\
&- g\, f_{abc}G^{aj}G^{bk}\partial_j G^{ck} \\
&- \frac{1}{4}g^2 f_{abc}f_{ars}G^{bj}G^{ck}G^{rj}G^{sk} \\
&+ \sum_q\left\{\bar{q}\frac{\mathrm{i}}{2}\gamma^0\dot{q}\right. \qquad \text{(C.3)} \\
&\left.- \dot{\bar{q}}\frac{\mathrm{i}}{2}\gamma^0 q + \bar{q}\left(\frac{\mathrm{i}}{2}\gamma^j\overleftrightarrow{\partial}_j - m_q + g\gamma^j G^{aj}\frac{\lambda_a}{2}\right)q\right\}.
\end{aligned}$$

The canonical momenta are

$$\Pi_{G^{aj}} = \frac{\partial \tilde{\mathcal{L}}}{\partial \dot{G}^{aj}} = \dot{G}^{aj} \quad \text{(C.4)}$$

and $\Pi_q, \Pi_{\bar{q}}$ as in (B.4). This leads to the following Hamiltonian density and 03 component of the canonical energy-momentum tensor:

$$\begin{aligned} \mathcal{H} = & \frac{1}{2}\Pi_{G^{aj}}\Pi_{G^{aj}} \\ & + \frac{1}{2}(\partial_k G^{aj})\partial_k G^{aj} - \frac{1}{2}(\partial_j G^{aj})\partial_k G^{ak} \\ & + g\, f_{abc} G^{aj} G^{bk} \partial_j G^{ck} \\ & + \frac{1}{4} g^2 f_{abc} f_{ast} G^{bj} G^{ck} G^{sj} G^{tk} \\ & + \sum_q \left\{ \bar{q}\left(-\frac{\mathrm{i}}{2}\gamma^j \overleftrightarrow{\partial}_j + m_q - g\gamma^j G^{aj}\frac{\lambda_a}{2} \right) q \right\}, \quad \text{(C.5)} \end{aligned}$$

$$\mathcal{T}_{03} = \Pi_{G^{aj}}\partial_3 G^{aj} + \sum_q \left\{ \bar{q}\frac{\mathrm{i}}{2}\gamma^0 \overleftrightarrow{\partial}_3\, q \right\}. \quad \text{(C.6)}$$

With this we get for the r-dependent Hamiltonian density

$$\begin{aligned} \mathcal{H}_r &= \mathcal{H} + (1-r)\mathcal{T}_{03} \\ &= \mathcal{H}_r^{(0)} + \mathcal{H}^{\rm Int}, \quad \text{(C.7)} \\ \mathcal{H}_r^{(0)} &= \frac{1}{2}\Pi_{G^{aj}}\Pi_{G^{aj}} + (1-r)\Pi_{G^{aj}}\partial_3 G^{aj} \\ & + \frac{1}{2}(\partial_j G^{ak})\partial_j G^{ak} - \frac{1}{2}(\partial_j G^{aj})\partial_k G^{ak} \quad \text{(C.8)} \\ & + \sum_q \left\{ \bar{q}\left(-\frac{\mathrm{i}}{2}\gamma^j \overleftrightarrow{\partial}_j + (1-r)\frac{\mathrm{i}}{2}\gamma^0 \overleftrightarrow{\partial}_3 + m_q \right) q \right\}, \\ \mathcal{H}^{\rm Int} &= g\, f_{abc} G^{aj} G^{bk}\partial_j G^{ck} + \frac{1}{4} g^2 f_{abc} f_{ast} G^{bj} G^{ck} G^{sj} G^{tk} \\ & - \sum_q \left\{ \bar{q} g \gamma^j G^{aj} \frac{\lambda_a}{2} q \right\}. \quad \text{(C.9)} \end{aligned}$$

In the standard way we find from (C.8) and (C.9) the r-dependent Lagrangian density

$$\begin{aligned} \mathcal{L}_r &= \mathcal{L}_r^{(0)} + \mathcal{L}^{\rm Int}, \quad \text{(C.10)} \\ \mathcal{L}_r^{(0)} &= \frac{1}{2}(\dot{G}^{aj} - (1-r)\partial_3 G^{aj})(\dot{G}^{aj} - (1-r)\partial_3 G^{aj}) \\ & - \frac{1}{2}(\partial_k G^{aj})\partial_k G^{aj} + \frac{1}{2}(\partial_j G^{aj})\partial_k G^{ak} \quad \text{(C.11)} \\ & + \sum_q \left\{ \bar{q}\left(\frac{\mathrm{i}}{2}\gamma^\lambda \overleftrightarrow{\partial}_\lambda - (1-r)\frac{\mathrm{i}}{2}\gamma^0 \overleftrightarrow{\partial}_3 - m_q \right) q \right\}, \\ \mathcal{L}^{\rm Int} &= -\mathcal{H}^{\rm Int}. \quad \text{(C.12)} \end{aligned}$$

We also find with $\tilde{\mathcal{L}}$ from (C.3).

$$\begin{aligned} \mathcal{L}_r = & \tilde{\mathcal{L}} - (1-r)\dot{G}^{aj}\partial_3 G^{aj} + \frac{1}{2}(1-r)^2(\partial_3 G^{aj})\partial_3 G^{aj} \\ & - (1-r)\sum_q \left\{ \bar{q}\frac{\mathrm{i}}{2}\gamma^0 \overleftrightarrow{\partial}_3\, q \right\}. \quad \text{(C.13)} \end{aligned}$$

D Euclidean Lagrangian density in covariant gauges

Let H_r be as in (4.12); to be precise, we set $x^0 = 0$:

$$H_r = \int \mathrm{d}^3 x \mathcal{H}_r(x)|_{x^0=0}. \quad \text{(D.1)}$$

We define the Euclidean field operators with

$$V(X_4) = \exp(X_4 H_r) \quad \text{(D.2)}$$

as follows:

$$\begin{aligned} G^a_{\mathrm{E}j}(X) &= -V(X_4) G^{aj}(x) V(-X_4), \\ G^a_{\mathrm{E}4}(X) &= -\mathrm{i} V(X_4) G^{a0}(x) V(-X_4), \\ q_{\mathrm{E}}(X) &= V(X_4) q(x) V(-X_4), \\ \bar{q}_{\mathrm{E}}(X) &= V(X_4)\bar{q}(x) V(-X_4), \\ \phi^a_{\mathrm{E}}(X) &= V(X_4)\phi^a(x) V(-X_4), \\ \bar{\phi}^a_{\mathrm{E}}(X) &= V(X_4)\bar{\phi}^a(x) V(-X_4), \\ \Pi_{G^a_{\mathrm{E}j}}(X) &= -V(X_4)\Pi_{G^{aj}}(x) V(-X_4), \\ \Pi_{G^a_{\mathrm{E}4}}(X) &= \mathrm{i} V(X_4)\Pi_{G^{a0}}(x) V(-X_4), \\ \Pi_{\phi^a_{\mathrm{E}}}(X) &= V(X_4)\Pi_{\phi^a}(x) V(-X_4), \\ \Pi_{\bar{\phi}^a_{\mathrm{E}}}(X) &= V(X_4)\Pi_{\bar{\phi}^a}(x) V(-X_4). \quad \text{(D.3)} \end{aligned}$$

Here $G^{aj}(x)$ etc. on the r.h.s. of (D.3) are the Minkowskian field operators at the point

$$x = (\mathbf{X}, 0), \quad \text{(D.4)}$$

and $X = (\mathbf{X}, X_4)$.

Defining the Euclidean electromagnetic current operator as

$$J_{\mathrm{E}\mu}(X) = \sum_q \left\{ Q_q \bar{q}_{\mathrm{E}}(X)\gamma_{\mathrm{E}\mu} q_{\mathrm{E}}(X) \right\}, \quad \text{(D.5)}$$

where Q_q are the quark charges, we get with x as in (D.4)

$$\begin{aligned} J_{\mathrm{E}4}(X) &= V(X_4) J^0(x) V(-X_4), \\ J_{\mathrm{E}j}(X) &= -\mathrm{i} V(X_4) J^j(x) V(-X_4). \quad \text{(D.6)} \end{aligned}$$

The nucleon field operators in the Euclidean theory are obtained from (4.16):

$$\begin{aligned} A_{\mathrm{E}s}(p,\tau) &= V(\tau) A_s(p,0) V(-\tau), \\ A^\dagger_{\mathrm{E}s}(p,\tau) &= V(\tau) A^\dagger_s(p,0) V(-\tau) \quad \left(s = \pm\frac{1}{2}\right), \quad \text{(D.7)} \end{aligned}$$

where we always take p (2.18) for a nucleon at rest. We have

$$\begin{aligned} \lim_{\tau\to-\infty} A^\dagger_{\mathrm{E}s}(p,\tau) \mid 0\rangle \exp(-M\tau) &= \mid N(p,s)\rangle, \\ \lim_{\tau\to\infty} \langle 0 \mid A_{\mathrm{E}s}(p,\tau)\exp(M\tau) &= \langle N(p,s) \mid . \quad \text{(D.8)} \end{aligned}$$

In the standard way we get now for the matrix elements $\widetilde{\mathcal{M}}^-_{a,b}(-\mathrm{i}X_4, r)$ defined in (4.7) and (4.8), for $X_4 > 0$,

$$\begin{aligned}\widetilde{\mathcal{M}}^-_a(-\mathrm{i}X_4, r) &= \frac{1}{2}\sum_s \langle N(p,s) \mid (-1) \\ &\times J_{\mathrm{E}\mu}(\mathbf{0}, X_4) J_{\mathrm{E}\mu}(\mathbf{0},0) \mid N(p,s)\rangle \\ &= \frac{1}{2}\sum_s \lim_{\substack{\tau_i\to-\infty \\ \tau_f\to+\infty}} \exp[(\tau_f-\tau_i)M] \qquad \text{(D.9)} \\ &\times \langle 0 \mid A_s(p,\tau_f)(-1) J_{\mathrm{E}\mu}(\mathbf{0}, X_4) J_{\mathrm{E}\mu}(\mathbf{0},0) A_s^\dagger(p,\tau_i) \mid 0\rangle.\end{aligned}$$

$$\begin{aligned}&\widetilde{\mathcal{M}}^-_b(-\mathrm{i}X_4, r) \\ &= \frac{1}{2}\sum_s \langle N(p,s) \mid J_{\mathrm{E}4}(\mathbf{0}, X_4) J_{\mathrm{E}4}(\mathbf{0},0) \mid N(p,s)\rangle \\ &= \frac{1}{2}\sum_s \lim_{\substack{\tau_i\to-\infty \\ \tau_f\to+\infty}} \exp[(\tau_f-\tau_i)M] \qquad \text{(D.10)} \\ &\times \langle 0 \mid A_s(p,\tau_f) J_{\mathrm{E}4}(\mathbf{0}, X_4) J_{\mathrm{E}4}(\mathbf{0},0) A_s^\dagger(p,\tau_i) \mid 0\rangle.\end{aligned}$$

The Euclidean Hamiltonian density is obtained from (4.13) with x from (D.4)

$$\mathcal{H}_{\mathrm{E}r}(X) = V(X_4)\mathcal{H}_r(x)V(-X_4). \qquad \text{(D.11)}$$

Using here (D.3) leads to

$$\begin{aligned}\mathcal{H}_{\mathrm{E}r}(X) &= \frac{1}{2}\xi \Pi_{G^a_{\mathrm{E}4}}\Pi_{G^a_{\mathrm{E}4}} + \frac{1}{2}\Pi_{G^a_{\mathrm{E}j}}\Pi_{G^a_{\mathrm{E}j}} \\ &- \mathrm{i}\Pi_{G^a_{\mathrm{E}4}}(\partial_j G^a_{\mathrm{E}j} + \mathrm{i}(1-r)\partial_3 G^a_{\mathrm{E}4}) \\ &+ \Pi_{G^a_{\mathrm{E}j}}(\mathrm{i}\partial_j G^a_{\mathrm{E}4} + \mathrm{i}g f_{abc} G^b_{\mathrm{E}4} G^c_{\mathrm{E}j} + (1-r)\partial_3 G^a_{\mathrm{E}j}) \\ &+ \frac{1}{4} G^a_{\mathrm{E}jk} G^a_{\mathrm{E}jk} \\ &+ \sum_q \left\{ \bar{q}_\mathrm{E} \left(\frac{1}{2}\gamma_{\mathrm{E}j} \overleftrightarrow{\partial}_j + \frac{\mathrm{i}}{2}\gamma_{\mathrm{E}4}(1-r) \overleftrightarrow{\partial}_3 \right.\right. \\ &\left.\left. + \ \mathrm{i}g\gamma_{\mathrm{E}\mu} G^a_{\mathrm{E}\mu}\frac{1}{2}\lambda_a + m_q \right) q_\mathrm{E} \right\} \\ &+ \Pi_{\phi^a_\mathrm{E}}\Pi_{\bar{\phi}^a_\mathrm{E}} + \Pi_{\phi^a_\mathrm{E}}(\mathrm{i}g f_{abc} G^b_{\mathrm{E}4}\phi^c_\mathrm{E} + (1-r)\partial_3\phi^a_\mathrm{E}) \\ &+ (1-r)(\partial_3\bar{\phi}^a_\mathrm{E})\Pi_{\bar{\phi}^a_\mathrm{E}} + (\partial_j\bar{\phi}^a_\mathrm{E})\partial_j\phi^a_\mathrm{E} \\ &- g f_{abc}(\partial_j\bar{\phi}^a_\mathrm{E}) G^b_{\mathrm{E}j}\phi^c_\mathrm{E}, \qquad \text{(D.12)}\end{aligned}$$

where all fields on the r.h.s. are at the point X and we define the Euclidean field strength tensor by

$$G^a_{\mathrm{E}\mu\nu} = \partial_\mu G^a_{\mathrm{E}\nu} - \partial_\nu G^a_{\mathrm{E}\mu} - g f_{abc} G^b_{\mathrm{E}\mu} G^c_{\mathrm{E}\nu}. \qquad \text{(D.13)}$$

It is now straightforward to verify that $\mathcal{H}_{\mathrm{E}r}$ is the Euclidean Hamiltonian density to the Lagrangian density $\mathcal{L}_{\mathrm{E}r}$ of (4.22). Indeed, starting from (4.22) we define the Euclidean canonical momenta as follows, where $\dot{G}^a_{\mathrm{E}4} \equiv \partial_4 G^a_{\mathrm{E}4}$ and so on:

$$\begin{aligned}\mathrm{i}\frac{\partial\mathcal{L}_{\mathrm{E}r}}{\partial\dot{G}^a_{\mathrm{E}4}} &= \Pi_{G^a_{\mathrm{E}4}} \\ &= \frac{1}{\xi}\{\mathrm{i}\dot{G}^a_{\mathrm{E}4} + \mathrm{i}\partial_j G^a_{\mathrm{E}j} - (1-r)\partial_3 G^a_{\mathrm{E}4}\}, \\ \mathrm{i}\frac{\partial\mathcal{L}_{\mathrm{E}r}}{\partial\dot{G}^a_{\mathrm{E}j}} &= \Pi_{G^a_{\mathrm{E}j}} \\ &= \mathrm{i}\dot{G}^a_{\mathrm{E}j} - \mathrm{i}\partial_j G^a_{\mathrm{E}4} - \mathrm{i}g f_{abc} G^b_{\mathrm{E}4} G^c_{\mathrm{E}j} - (1-r)\partial_3 G^a_{\mathrm{E}j}, \\ \mathrm{i}\frac{\partial\mathcal{L}_{\mathrm{E}r}}{\partial\dot{\phi}^a_\mathrm{E}} &= \Pi_{\phi^a_\mathrm{E}} \\ &= \mathrm{i}\dot{\bar{\phi}}^a_\mathrm{E} - (1-r)\partial_3\bar{\phi}^a_\mathrm{E}, \\ \mathrm{i}\frac{\partial\mathcal{L}_{\mathrm{E}r}}{\partial\dot{\bar{\phi}}^a_\mathrm{E}} &= \Pi_{\bar{\phi}^a_\mathrm{E}} \\ &= \mathrm{i}\dot{\phi}^a_\mathrm{E} - \mathrm{i}g f_{abc} G^b_{\mathrm{E}4}\phi^c_\mathrm{E} - (1-r)\partial_3\phi^a_\mathrm{E}, \\ \mathrm{i}\frac{\partial\mathcal{L}_{\mathrm{E}r}}{\partial\dot{q}_\mathrm{E}} &= \Pi_{q_\mathrm{E}} = \bar{q}_\mathrm{E}\frac{\mathrm{i}}{2}\gamma_{\mathrm{E}4}, \\ \mathrm{i}\frac{\partial\mathcal{L}_{\mathrm{E}r}}{\partial\dot{\bar{q}}_\mathrm{E}} &= \Pi_{\bar{q}_\mathrm{E}} = -\frac{\mathrm{i}}{2}\gamma_{\mathrm{E}4} q_\mathrm{E}. \qquad \text{(D.14)}\end{aligned}$$

The Hamiltonian density (D.12) is now obtained from (4.22) and (D.14) as

$$\begin{aligned}\mathcal{H}_{\mathrm{E}r} &= \mathrm{i}\Pi_{G^a_{\mathrm{E}\mu}}\dot{G}_{\mathrm{E}\mu} + \mathrm{i}\sum_q(\dot{\bar{q}}_\mathrm{E}\Pi_{\bar{q}_\mathrm{E}} + \Pi_{q_\mathrm{E}}\dot{q}_\mathrm{E}) \\ &+ \mathrm{i}\Pi_{\phi^a_\mathrm{E}}\dot{\phi}^a_\mathrm{E} + \mathrm{i}\dot{\bar{\phi}}^a_\mathrm{E}\Pi_{\bar{\phi}^a_\mathrm{E}} + \mathcal{L}_{\mathrm{E}r}, \qquad \text{(D.15)}\end{aligned}$$

where the X_4 derivatives $\dot{G}_{\mathrm{E}\mu}$ etc. have to be considered as functions of the canonical momenta and the fields by inverting (D.14).

Having derived the connection of the Euclidean Hamilton and Lagrange densities (D.12) and (4.22) we can use the standard procedures of the path integral formalism to show that the matrix elements (D.9) and (D.10) can be represented by the path integrals (4.26) and (4.27).

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